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ABSTRACT

The objective of the Reactor Dynamics Module, RD-1, is to obtain the kinetics equation without feedback and solve the kinetics equations numerically for one to six delayed neutron groups for time varying reactivity insertions. The computer code FUNCKI (Fundamental Mode Kinetics) will calculate the power as a function of time for either uranium or plutonium. Either fuel can be used with one to six delayed neutron groups and one of three types of reactivity insertions: a constant reactivity, sinuscidal, or a ramp. The code does not compute any parameters so the neutron generation time must be provided. The user has the option of studying the effects of various time steps in solving the system. The objective of Module RD-2 is to examine the temperature feedback mechanism of a pressurized water reactor (PWR) and solve the one delayed neutron model with temperature feedback for a step insertion and a ramp insertion of reactivity. A PWR core with a two-path feedback is considered. The core region is the only one of interest in this module. The program name is FUMOTEM (Fundamental Mode Kinetics with Temperature Feedback). There are four types of reactivity inputs that the program can accommodate. (Author)

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REACTOR DYNAMICS MODULE, RD-1

THE REACTOR KINETICS EQUATIONS

by

Ronald J. Onega

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KINETICS MODULE 1

THE REACTOR KINETICS EQUATIONS

1.1 Object of Module

The object of this module is to:

1) Obtain the Kinetics Equation without feedback

and 2) Solve the kinetics equations numerically for one to six delayed neutron groups for time varying reactivity insertions.

The time dependence of a modern reactor is really very complicated. The control rod motion is a local perturbation so the time dependence of the flux cannot completely be divorced from the space dependence. The fundamental mode kinetics equations do provide a rough idea of what the time behavior of a reactor will be. In this module we will develop the kinetics equations and indicate how they can be solved numerically. Feedback effects will be introduced in later modules.

The computer code FUMOKI (Fundamental Mode Kinetics) will calculate the power as a function of time for either uranium or plutonium. Either fuel can be used with one to six delayed neutron groups and one of three types of reactivity insertions:

1) a constant reactivity ρ_o

2) sinusoidal $\rho(t) = \rho_o \sin b_2 t$

or 3) a ramp $\rho(t) = \rho_o(1 + b_3 t)$.

The ramp is to simulate the rod withdrawl or insertion reactivity input.

The code does not compute any parameters so the neutron generation time must be provided. Also the user has the option of studying the effects of various time steps in solving the system. The time step can be surprisingly large (0.05 sec) and still yield good results in most cases.

1.2 The Kinetics Equations

The time behavior of a reactor is a very important consideration in the operation of a nuclear power plant. Also, the safety analysis of a plant depends upon a thorough knowledge of the kinetics equations. The many types of reactor designs necessitates consideration of various reactivity coefficients and dynamic response characteristics.

The neutronic considerations of a reactor cannot be divorced from associated feedback mechanisms such as heat transport, fluid flow, mechanical changes etc. There are many ways to delineate the dynamics problems of a power plant but a natural one seems to be to classify problems according to the time constants involved. There are four areas that we can study:

1) Very slow transients - fuel depletion with time constants of the order of a year or so. We will not consider this as a dynamics problem at all but rather in the statics sections.
2) Slow transients - Xenon and Samarium effects. The time constants here are of the order of hours.
3) Normal transients - Changes in fuel and moderator temperature, void changes, delayed neutron considerations etc. The time constants here are of the order of a second or so and small reactivity changes are involved.
4) Fast transients - Control rod dropped in or withdrawn at its maximum rate etc. Reactivity inputs are of 50¢ and up. The time constants are of the order of 10^{-4} sec. and serious safety questions are raised.

In addition to these rather general time reference frames, another important aspect of the reactor dynamics problem is whether or not the core is so large that spatially dependent analysis is necessary. The solution of one or more dimensional kinetics problems necessitates the use of a relatively large computer. Xenon oscillations are of importance here. Also reactivity insertions are usually localized so that hot spots may develop.

Coupled core kinetics considerations are of interest in some types of reactors with large cores or core regions which are loosely coupled. When the neutron flight times between different regions of the reactor are not negligible, then coupled core kinetics may be a useful tool. Coupled core kinetics equations are generally differential-integral equations with time-lag kernels. Coupled core kinetics involves writing the kinetics equations for each region of the reactor and then coupling the regions by neutron leakage from one region to the other.

When a control rod in a reactor is removed, the neutron flux distribution is disturbed so that not only the fundamental mode is present but the higher modes are also present. However, these higher harmonics die out very rapidly so that while the rod is in motion (in the order of seconds) the fundamental mode is the only important one. Therefore we will deal only with the fundamental mode for the space dependence and the kinetics equations we solve will simply indicate how the amplitude of this fundamental mode changes with time. For example, in a spherical reactor the flux is

$$\phi(r,t) = \phi_0(t) \frac{\sin Br}{Br} ,$$

and this spatial dependence persists through most transients, and the kinetics equations simply yield the $\phi_o(t)$. In a word, the spatial and time dependence are separable for most transients.

We will derive the kinetics equations using one group diffusion theory but the same equations are obtained from multi-group or transport theory. The thermal neutron diffusion equation is

$$D \nabla^2 \phi - \Sigma_a \phi + S = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad . \tag{1.2.1}$$

The source is composed of three terms, i.e.

$$S = S_p + S_D + S_{ext} \quad , \tag{1.2.2}$$

where S_p = prompt neutron source,

S_D = delayed neutron source,

and S_{ext} = a source of neutrons entered externally, as from a Plutonium-Beryllium source.

Not all neutrons are emitted immediately from the fission process. The prompt neutrons are emitted within 10^{-10} sec. of the fission process itself but there are other neutrons called delayed neutrons. They arise from the beta decay of some of the fission fragments. As an example, Kr-93 is a fission fragment which has a half life of 1.22 sec. This beta-decays to Rb-93, but it is formed in such a highly excited state that a neutron is emitted following the beta decay and the reaction is thus,

$$^{93}Kr \xrightarrow[1.22 \text{ sec.}]{\beta} {}^{93}Rb \xrightarrow[\sim 10^{-16} \text{ sec.}]{} {}^{92}Rb + {}^{1}_{0}n \, .$$

There are some 30 of these neutron precursors (^{93}Kr) which produce neutrons

through beta decay.

Since the half-lives of many of these precursors are very close to each other, it is necessary only to consider six delayed neutron precursor groups which are averages of the 30 or so precursors, appropriately weighted. The delayed neutron precursors we talk about are thus fictitious in that they do not actually exist, but are averages of the actual precursors. Table 1.5.1 (section 5) indicates the groups of delayed neutrons obtained from these six precursors for neutron induced fission in uranium and plutonium.

The fraction of neutrons which are delayed is called β. For ^{235}U, $\beta = 0.0064$ and the total delayed neutron fraction is the sum of the individual ones, i.e.,

$$\beta = \sum_{i=1}^{6} \beta_i . \qquad (1.2.3)$$

Returning now to the diffusion equation, we can set

$$S_p = (1-\beta)\ \nu\Sigma_f\ \phi(\vec{r}, t) \qquad (1.2.4)$$

and

$$S_D = \sum_{i=1}^{6} \lambda_i\ C_i\ (\vec{r}, t) \qquad (1.2.5)$$

where $C_i\ (\vec{r}, t)$ is the precursor concentration of group "i", i.e., the number of precursors/cm^3 existing at point $\vec{r}$ at time t. Each time a precursor C_i decays, it is assumed a delayed neutron of group "i" results.

So our diffusion equation becomes (Equation 1.2.1)

$$D\ \nabla^2 \phi - \Sigma_a \phi + (1-\beta)\ \nu\Sigma_f \phi + \sum_{i=1}^{6} \lambda_i\ C_i\ (\vec{r}, t) = \frac{1}{v} \frac{\partial \phi}{\partial t} \qquad (1.2.6)$$

Equation(1.2.6)has four independent variables, x, y, z and t, and seven dependent variables ϕ, C_1, C_2, ..., C_6. In order to solve such a system of equations, we will have to write six more equations, one for each of the precursor concentrations. The rate at which the precursor concentration changes is

$$\frac{\partial C_i(\vec{r}, t)}{\partial t} = \underbrace{\beta_i \nu\Sigma_f \phi(\vec{r}, t)}_{\text{rate of formation}} - \underbrace{\lambda_i C_i(\vec{r}, t)}_{\text{rate of decay}}. \qquad (1.2.7)$$

$$i = 1, 2, \ldots, 6.$$

This system of equations is relatively difficult to solve so we shall work analytically with Equation (1.2.6) and (1.2.7) for a while.

Assume that we want to expand the spatial dependence of the flux $\phi(\vec{r}, t)$ and the precursor concentrations $C_i(\vec{r}, t)$ in terms of a set of eigenfunctions of the Helmholtz equation

$$\nabla^2 Y_n(\vec{r}) + B_n^2 Y_n(\vec{r}) = 0. \qquad (1.2.8)$$

The reason for doing this is that the solution for the steady state satisfies this same equation with coefficients which are time independent. In the time dependent problems the coefficients are functions of time. The eigenfunctions $Y_n(\vec{r})$ are Cos B_n x for a slab reactor, Bessel's functions for a cylindrical reactor, etc. In any event, we set

$$\phi(\vec{r}, t) = \sum_{n=0}^{\infty} \bar{\phi}_n(t) Y_n(\vec{r}) \qquad (1.2.9)$$

and

$$C_i\ (\vec{r},\ t) = \sum_{n=0}^{\infty} \tilde{C}_{in}\ (t)\ Y_n\ (\vec{r}). \qquad (1.2.10)$$

We note that $\tilde{\phi}_n\ (t)$ will yield the amplitude of the n^{th} harmonic and $Y_n\ (\vec{r})$ its spatial distribution. Substituting these expressions into Equations (1.2.6) and (1.2.7) and using Equation (1.2.8) we obtain

$$\sum_{n=0}^{\infty} \left[-DB_n^2\ \tilde{\phi}_n(t) - \Sigma_a \tilde{\phi}_n(t) + (1-\beta)\ \nu\Sigma_f\ \tilde{\phi}_n\ (t) \right] Y_n\ (\vec{r}) + \sum_{i=1}^{6} \sum_{n=0}^{\infty} \lambda_i\ \tilde{C}_{in}\ (t)\ \ Y_n\ \ (\vec{r}) = \frac{1}{v} \sum_{n=0}^{\infty} Y_n\ \ (\vec{r})\ \frac{d\tilde{\phi}_n(t)}{dt}$$

and

$$\sum_{n=0}^{\infty} Y_n\ (\vec{r})\ \ \frac{d\tilde{C}_{in}(t)}{dt} = \sum_{n=0}^{\infty} \left[\beta_i\ \nu\Sigma_f\ \tilde{\phi}_n\ (t) - \lambda_i\ \tilde{C}_{in}\ (t) \right] Y_n\ (\vec{r}).$$

Now using the fact that the $Y_n\ (\vec{r})$ form an orthogonal set, the preceding equation is simplified by multiplying it by $Y_m\ (\vec{r})$ and integrating over the reactor volume (taking the inner product) to yield

$$\frac{1}{v}\ \ \frac{d\tilde{\phi}_n}{dt}\ (t) = -(DB_n^2 + \Sigma_a)\ \tilde{\phi}_n\ (t) + (1-\beta)\ \nu\Sigma_f\ \tilde{\phi}_n\ (t) + \sum_{i=1}^{6} \lambda_i\ \tilde{C}_{in}\ (t), \qquad (1.2.11)$$

and

$$\frac{d\tilde{C}_{i_n}\ (t)}{dt} = \beta_i\ \nu\Sigma_f\ \tilde{\phi}_n\ (t) - \lambda_i\ \tilde{C}_{in}\ (t). \qquad (1.2.12)$$

Now it is necessary to solve these two equations for the expansion coefficients $\tilde{\phi}_n\ (t)$ and $\tilde{C}_{in}\ (t)$. This is a very difficult task so we

make some definitions and some approximations.

First we define the multiplication k_n as

$$k_n = \frac{\nu \Sigma_f / \Sigma_a}{1 + B_n^2 L^2} \tag{1.2.13}$$

and the thermal neutron lifetime

$$\ell_n = \frac{1}{\bar{v} \Sigma_a (1 + B_n^2 L^2)} \cdot \tag{1.2.14}$$

The next thing we do is assume that there are no delayed neutrons (there are, but assume for the moment we can neglect them). Then Equation (1.2.11) becomes (if $\beta = 0$),

$$\frac{d\tilde{\phi}_n}{dt}(t) = -v \Sigma_a (1 + L^2 B_n^2) \tilde{\phi}_n(t) + v \nu \Sigma_f \tilde{\phi}_n(t).$$

Now using Equation (1.2.14) we have

$$\frac{d\tilde{\phi}_n}{dt}(t) = -\frac{\tilde{\phi}_n}{\ell_n}(t) + v \nu \frac{\Sigma_f}{\Sigma_{au}} \frac{\Sigma_{au}}{\Sigma_a} \Sigma_a \tilde{\phi}_n(t),$$

$$= -\frac{\tilde{\phi}_n}{\ell_n}(t) + v \Sigma_a \cdot k_\infty \cdot \frac{1 + B_n^2 L^2}{1 + B_n^2 L^2} \phi_n(t),$$

$$= \frac{k_n - 1}{\ell_n} \phi_n(t). \tag{1.2.15}$$

The solution of Equation (1.2.15) is obviously

$$\tilde{\phi}_n(t) = \tilde{\phi}_n(o) \, e^{\frac{k_n - 1}{\ell_n} t} \tag{1.2.16}$$

For a typical light water reactor, $\ell_o = \ell = 10^{-4}$ sec. and $k_o = k = 1.0020$. From Equation (1.2.14) we note that

$$\frac{\ell_n}{\ell} = \frac{1 + B_o^2 L^2}{1 + B_n^2 L^2} \sim \frac{1}{n^2}$$

so that the higher harmonics die out rapidly. We will keep only the fundamental mode, not only in the solution of the no delayed neutron case but also for the solution of Equations (1.2.11) and (1.2.12).

Returning now to Equations (1.2.11) and (1.2.12) and omitting the subscript n since we are concerned only with the fundamental mode, we set

$$\tilde{\phi}(t) = v\, n(t)$$

where n (t) is the neutron density (cm^{-3}), and after a bit of algebra come up with

$$\frac{dn(t)}{dt} = (1-\beta)\, \nu \frac{\Sigma_f}{\Sigma_a}\, v\, \Sigma_a\, n(t) - v\, \Sigma_a\, (1 + B^2 L^2)\, n(t) + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t)$$

and

$$\frac{d\tilde{C}_i(t)}{dt} = \beta_i\, v\, \Sigma_a\, \nu \frac{\Sigma_f}{\Sigma_a}\, n(t) - \lambda_i \tilde{C}_i(t).$$

Using the definitions given in Equations (1.2.13) and (1.2.14) the above equations become

$$\frac{dn(t)}{dt} = (i - \beta) \frac{k}{\ell} n - \frac{n}{\ell} + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t)$$

and

$$\frac{d\tilde{C}_i(t)}{dt} = \beta_i \frac{k}{\ell} n(t) - \lambda_i \tilde{C}_i(t).$$

k is the effective multiplication, and the effective neutron lifetime is denoted as ℓ. If we drop the tilde on the C_i's, then we have

$$\frac{dn(t)}{dt} = \frac{(1-\beta)\,k - 1}{\ell} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \qquad (1.2.18)$$

and

$$\frac{dC_i(t)}{dt} = \beta_i \frac{k}{\ell} n(t) - \lambda_i C_i(t). \qquad (1.2.19)$$

These are called the fundamental mode reactor kinetics equations, in that spatial and time dependence are assumed separable.

Sometimes it's convenient to cast these equations into a slightly different form. The thermal neutron lifetime is

$$\ell = \frac{\lambda_a}{v} \frac{1}{1 + B^2 L^2} \qquad (1.2.20)$$

i.e., the mean time a neutron spends in the system from its birth as a thermal neutron until it's absorbed or leaks out of the reactor. The neutron gereration time is defined as

$$\Lambda = \frac{\ell}{k} = \frac{1}{v \Sigma_a (1 + B^2 L^2)} \cdot \frac{\Sigma_a (1 + B^2 L^2)}{v \Sigma_f} = \frac{1}{v \nu \Sigma_f}. \qquad (1.2.21)$$

The generation time is the mean time that it takes one neutron to generate one more prompt neutron or one precursor. The neutron lifetime is thus the reciprocal of the destruction rate of neutrons and the generation time is the reciprocal of the production rate of neutrons. With this definition and the definition of reactivity ρ,

$$\rho \equiv \frac{k-1}{k} , \tag{1.2.22}$$

the kinetics equations (1.2.18 and 1.2.19) become

$$\frac{dn}{dt}(t) = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i (t) \tag{1.2.23}$$

$$\frac{d C_i (t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i (t) \tag{1.2.24}$$

$$i = 1,2,\ldots,6$$

Equations (1.2.23) and (1.2.24) are used to describe the time behavior of a reactor. This form of the equations results if multigroup diffusion theory or transport theory is used to derive the equations but the definitions of ℓ, Λ, ρ, k are modified. We will assume they are input parameters to the system of equations.

Problem 1.2.1

Estimate the neutron lifetime and neutron generation time in an infinite stack of graphite and U-235 if $\sigma_a = 5 \times 10^{-3}$ b for the graphite and there are 500 atoms of carbon to each atom of U-235. There is no U-238 present. $\sigma_a = 680$ b for the U-235 and take v to be 2200 m/sec.

Problem 1.2.2

Estimate the period of a reactor if there are no delayed neutrons and if the k = 1.0010 and $\ell = 10^{-4}$ sec. The reactor period is the time it takes for the flux or neutron density to increase by a factor of e.

Problem 1.2.3

Let $Y_i = C_i(t) \frac{\Lambda \lambda_i}{\beta_i}$ and $\alpha_R = \beta/\Lambda$. Show that the kinetics equations are then

$$\frac{dn}{dt} = \alpha_R \left[n(t) \quad (\rho'-1) \quad + \sum_{i=1}^{6} a_i Y_i \right]$$

and

$$\frac{dY_i}{dt} = \lambda_i \left[n(t) - Y_i(t) \right]$$

where $\rho' = \frac{\rho(t)}{\beta}$ and $a_i = \beta_i/\beta$.

1.3 Analytical Solutions of the Reactor Kinetics Equations

The kinetics equations are relatively difficult to solve both analytically and numerically due to the large difference in the time constants in the equations as well as due to the fact that there are seven coupled ordinary differential equations if there are six delayed neutron groups.

Equations (1.2.23) and (1.2.24) are repeated here as

$$\frac{dn}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \qquad (1.3.1)$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\Lambda} - \lambda_i C_i(t) \qquad i = 1, 2, \ldots, 6. \qquad (1.3.2)$$

If we assume that we can take an appropriate average for the delayed neutron decay constant, then it is possible to collapse these seven equations into two equations. For one effective group of delayed neutrons, Equations (1.3.1) and (1.3.2) reduce to

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \lambda C(t) \qquad (1.3.3)$$

and

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda C(t). \qquad (1.3.4)$$

The decay constant λ for this average delayed neutron precursor $C(t)$ is

$$\frac{1}{\lambda} = \frac{1}{\beta} \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} . \qquad (1.3.5)$$

This average is not unique and at times it may be advantageous to use another expression for the average. If very small reactivities are added to the reactor, then instead of averaging over all six groups, perhaps only the longest three would be used.

If only one group is desired Equation (1.3.5) is used to determine the effective λ and β is simply the sum of the β_i. For two groups, we split the β's into two groups of three each and

$$\beta_1 = \sum_{i=1}^{3} \beta_i, \qquad \frac{1}{\lambda_1} = \frac{1}{\beta_1} \sum_{i=1}^{3} \frac{\beta_i}{\lambda_i}, \tag{1.3.5a}$$

and

$$\beta_2 = \sum_{i=4}^{6} \beta_i, \qquad \frac{1}{\lambda_2} = \frac{1}{\beta_2} \sum_{i=4}^{6} \frac{\beta_i}{\lambda_i}. \tag{1.3.5b}$$

For three groups, the β's are split into three groups of two components each and for four groups, the shortest two groups are averaged together as well as the next shortest two and the 22 and 55 sec. groups are treated separately.

We will solve Equations (1.3.3) and (1.3.4) subject to a constant reactivity insertion ρ_o at time zero. In order to do this we recast our equations into a matrix form because of the similarity to the numerical technique (Hansen's method) used in the solution of the equations. In matrix form, Equations (1.3.3) and (1.3.4) become

$$\frac{d\underline{\psi}(t)}{dt} = \underline{\underline{A}}\,\underline{\psi}(t) \tag{1.3.6}$$

with

$$\underline{\psi}(t) = \begin{bmatrix} n(t) \\ \\ C(t) \end{bmatrix} \qquad \text{and } \underline{\underline{A}} = \begin{bmatrix} \frac{\rho_o - \beta}{\Lambda} & \lambda \\ \\ \frac{\beta}{\Lambda} & -\lambda \end{bmatrix}.$$

Notice that the $\underline{\underline{A}}$ matrix is independent of time if the reactivity is a constant. Equation (1.3.6) can formally be integrated to yield

$$\underline{\psi}(t) = e^{\underline{\underline{A}}t}\, \underline{\psi}(0). \tag{1.3.7}$$

The initial condition vector $\underline{\psi}(0)$ is

$$\underline{\psi}(0) = \begin{bmatrix} n(0) \\ \\ C(0) \end{bmatrix} = n(0) \begin{bmatrix} 1 \\ \\ \frac{\beta}{\lambda\Lambda} \end{bmatrix} \tag{1.3.8}$$

where we have used Equation (1.3.4) and set the derivative $\frac{dC}{dt} = 0$ for $t \leq 0$. When reactivity is added at time zero, the delayed neutron precursor concentration C(t) does not change until after some time. It is assumed that the reactor has been operating for a long time at a reactor power consistent with a neutron density n(0).

The formal solution given by Equation (1.3.7) is not much good if the explicit functional dependence of n(t) and C(t) cannot be obtained. The reason no explicit functional dependence is achieved from Equation (1.3.7) is that matrix $\underline{\underline{A}}$ is not diagonal. This means we really haven't separated the equations from each other. In order to demonstrate how the solution can be obtained, we imitate the technique used for the solution of an

$n^{\underline{th}}$ order differential equation, i.e. set

$$\underline{\psi}(t) = e^{\omega t}\,\underline{v}\,, \tag{1.3.9}$$

where $\underline{v}$ is a constant vector and ω is a scalar independent of time. Substituting this into Equation (1.3.6) we have

$$\omega\, e^{\omega t}\,\underline{v} = \underline{\underline{A}}\, e^{\omega t}\,\underline{v}$$

or

$$\underline{\underline{A}}\,\underline{v} = \omega\,\underline{v}. \tag{1.3.10}$$

Now it is apparent that Equation (1.3.10) is an eigenvalue equation so

$$|\underline{\underline{A}} - \omega\underline{\underline{I}}| = 0 \tag{1.3.11}$$

must be satisfied to determine the eigenvalue ω. This equation is called the characteristic equation and in reactor dynamics it is called the "inhour equation" because it relates the reactivity inserted into $\underline{\underline{A}}$ with the ω which is the reciprocal of the reactor period.

The inhour equation (Equation 1.3.11) for the one group delayed neutron model is

$$\det \begin{bmatrix} \frac{\rho_o - \beta}{\Lambda} - \omega & \lambda \\ \frac{\beta}{\Lambda} & -\lambda - \omega \end{bmatrix} = 0 \tag{1.3.12}$$

or

$$\Lambda\,\omega^2 + (\beta - \rho_o + \Lambda\,\omega) - \lambda\,\rho_o = 0. \tag{1.3.13}$$

Equation (1.3.13) can also be written as

$$\rho_o = \Lambda\,\omega + \frac{\beta\omega}{\omega+\lambda} \,. \tag{1.3.14}$$

If we had kept all six groups of delayed neutrons, Equation (1.3.11) would be a 7 x 7 determinant and Equation (1.3.14) would be enlarged to

$$\rho_o = \Lambda\,\omega + \sum_{i=1}^{6} \frac{\beta_i\omega}{\omega + \lambda_i} \tag{1.3.15}$$

The $\underline{v}$ of Equation (1.3.10) is the eigenvector associated with the eigenvalue ω. Now if we perform a similarity transformation $\underline{\underline{B}}$ on $\underline{\underline{A}}$, then both $\underline{\underline{A}}$ and $\underline{\underline{B}}^{-1}\ \underline{\underline{A}}\ \underline{\underline{B}}$ have the same characteristic equation. Also the fact the fact that the trace of $\underline{\underline{A}}$ is the sum of the eigenvalues of the matrix A is a useful check on the actual calculation of the eigenvalues of A. From this last fact it is apparent that the sum of the two roots ω_1 and ω_2 (the two eigenvalues) of Equation (1.3.13) is given by the relation

$$\omega_1 + \omega_2 = \frac{\rho_o-\beta}{\Lambda} - \lambda \,. \tag{1.3.16}$$

The eigenvectors $\underline{v}_1$ and $\underline{v}_2$ associated with ω_1 and ω_2 respectively are obtained by taking the cofactors of the element in any <u>row</u> of Equation (1.3.12). To see how this works, we set

$$\underline{v}_1 = \begin{bmatrix} v_{11} \\ \\ v_{21} \end{bmatrix} \quad \text{and} \quad \underline{v}_2 = \begin{bmatrix} v_{12} \\ \\ v_{22} \end{bmatrix} ,$$

and have v_{11}, the cofactor of the element $a_{11} - \omega_1$, as

$$v_{11} = -(\lambda + \omega_1),$$

and the cofactor of the element λ of the first row as

$$v_{21} = -\beta/\Lambda .$$

Similarly, for the vector $\underline{v}_2$, we have for the second row of $|\underline{\underline{A}} - \underline{\underline{I}}\omega|$,

$$v_{12} = -\lambda , \qquad v_{22} = \frac{\rho_o - \beta}{\Lambda} - \omega_2 .$$

Notice that to get the $\underline{v}_1$, we use the eigenvalue ω_1 and for $\underline{v}_2$, we use the eigenvalue ω_2 in Equation (1.3.12). Therefore, the eigenvectors of $\underline{\underline{A}}$ are (to within a constant)

$$\underline{v}_1 = \begin{bmatrix} \lambda + \omega_1 \\ \\ \frac{\beta}{\Lambda} \end{bmatrix} \quad \text{and} \quad \underline{v}_2 = \begin{bmatrix} \lambda \\ \\ \frac{\beta - \rho_o}{\Lambda} + \omega_2 \end{bmatrix} .$$

These vectors $\underline{v}_1$ and $\underline{v}_2$ are linearly independent since the eigenvalues are distinct.

Now since we have

$$\underline{\underline{A}}\, \underline{v}_i = \omega_i \underline{v}_i \qquad \text{for} \quad i = 1, 2 ,$$

it is apparent that the similarity transformation

$$\underline{\underline{A}}\ \underline{\underline{B}} = \underline{\underline{B}}\ \underline{\underline{D}} \tag{1.3.17}$$

holds, where D is a diagonal matrix having the eigenvalues as its elements,

i.e.

$$\underline{\underline{D}} = \begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix}$$

and

$$\underline{\underline{B}} = [\, \underline{v}_1 \quad \underline{v}_2 \,] \, .$$

These two matrices are now completely determined for the one delayed neutron model.

If we make a transformation

$$\underline{\psi}(t) = \underline{\underline{B}}\, \underline{Z}(t), \tag{1.3.18}$$

then

$$\frac{d\underline{Z}(t)}{dt} = \underline{\underline{B}}^{-1}\, \underline{\underline{A}}\, \underline{\underline{B}}\, \underline{Z}(t)$$

$$= \underline{\underline{D}}\, \underline{Z}(t)$$

and its solution is obviously

$$\underline{Z}(t) = \begin{bmatrix} e^{\omega_1 t} & 0 \\ 0 & e^{\omega_2 t} \end{bmatrix} \underline{Z}(o) = e^{\underline{\underline{D}}t}\, \underline{Z}(o) \, . \tag{1.3.19}$$

Now using Equations (1.3.18) and (1.3.19) we have

$$\underline{\psi}(t) = \underline{\underline{B}}\, e^{\underline{\underline{D}}t}\, \underline{\underline{B}}^{-1}\, \underline{\psi}(o) \, . \tag{1.3.20}$$

This is the solution of the kinetics equations. Notice now that the exponential $e^{\underline{\underline{D}}t}$ is a diagonal matrix so the separation of the equations is effected.

We now write out the details of this procedure. The transformation matrix $\underline{\underline{B}}$ is

$$\underline{\underline{B}} = [\underline{v}_1 \quad \underline{v}_2] = \begin{bmatrix} \lambda + \omega & \lambda \\ \beta/\Lambda & \frac{\beta-\rho_o}{\Lambda} + \omega_2 \end{bmatrix}$$

and the determinant of $\underline{\underline{B}}$ is

$$\det \underline{\underline{B}} = \omega_1\omega_2 + \frac{\beta-\rho_o}{\Lambda}\omega_1 + \lambda\omega_2 - \frac{\lambda\rho_o}{\Lambda} . \qquad (1.3.21)$$

The inverse of $\underline{\underline{B}}$ is

$$\underline{\underline{B}}^{-1} = \frac{1}{\det \underline{\underline{B}}} \begin{bmatrix} \frac{\beta-\rho_o}{\Lambda} + \omega_2 & -\lambda \\ -\frac{\beta}{\Lambda} & \lambda + \omega_1 \end{bmatrix}$$

The solution of the one delayed neutron group equations is obtained from Equations (1.3.20) and (1.3.8)

$$\underline{\psi}(t) = \frac{n(o)}{\det \underline{\underline{B}}} \begin{bmatrix} \lambda + \omega_1 & \lambda \\ \frac{\beta}{\Lambda} & \frac{\beta-\rho_o}{\Lambda} + \omega_2 \end{bmatrix}$$

$$\times \begin{bmatrix} e^{\omega_1 t} & 0 \\ 0 & e^{\omega_2 t} \end{bmatrix}$$

$$\times \begin{bmatrix} \frac{\beta-\rho_o}{\Lambda} + \omega_2 & -\lambda \\ -\frac{\beta}{\Lambda} & \lambda + \omega_1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 \\ \frac{\beta}{\Lambda\lambda} \end{bmatrix}$$

$$= \frac{n(t)}{\text{Det } \underline{B}} \begin{bmatrix} (\omega_2 - \frac{\rho_o}{\Lambda})(\lambda + \omega_1)\, e^{\omega_1 t} + \frac{\omega_1 \beta}{\Lambda}\, e^{\omega_2 t} \\ \frac{\beta}{\Lambda}\,(\omega_2 - \frac{\rho_c}{\Lambda})\, e^{\omega_1 t} + (\frac{\beta-\rho_o}{\Lambda} + \omega_2)\, \frac{\omega_1 \beta}{\lambda\Lambda}\, e^{\omega_2 t} \end{bmatrix}. \qquad (1.3.23)$$

Even though Equations (1.3.23) represent the exact solution of the problem, we are generally not concerned with c(t) so we look only at the n(t) equation. Also from Equation (1.3.13) we have

$$\omega_{1,2} = \frac{-(\beta-\rho_o + \Lambda\lambda) \pm \sqrt{(\beta-\rho_o + \Lambda\lambda)^2 + 4\,\lambda\Lambda\,\rho_c}}{2\,\Lambda}. \qquad (1.3.24)$$

Note too that

$$\omega_1 \omega_2 = - \frac{\lambda \rho_o}{\Lambda} \tag{1.3.25}$$

and in agreement with Equation (1.3.16),

$$\omega_1 + \omega_2 = - \frac{\beta - \rho_o + \lambda \Lambda}{\Lambda} \ . \tag{1.3.26}$$

Now if we approximate ω_1 (the positive sign) and ω_2 (the negative sign) in Equation (1.3.24) by assuming that

$$(\beta - \rho_o + \lambda \Lambda)^2 >> 4 \lambda \rho_o \Lambda \ ,$$

which is true for most reactors, we have the radical of Equation (1.3.24) equal to

$$(\beta - \rho_o + \Lambda \lambda) \sqrt{1 + \frac{4\lambda \rho_o \Lambda}{(\beta - \rho_o + \lambda \Lambda)^2}}$$

or

$$\left[1 + \frac{1}{2} \frac{4\lambda \Lambda \rho_o}{(\beta - \rho_o + \lambda \Lambda)^2} + \cdots \right] (\beta - \rho_o + \lambda \Lambda) \ .$$

When this is used we obtain

$$\omega_1 \simeq \frac{\lambda \rho_o}{\beta - \rho_o} \quad \text{and} \quad \omega_2 = - \frac{\beta - \rho_o}{\Lambda} \ . \tag{1.3.27}$$

Note that if ρ_o is positive, then ω_1 is also positive.

The determinant of $\underline{B}$ is involved in the solution of n(t) so with the above approximations as well as by using Equation (1.3.25) we have

$$\det \underline{B} \doteq -\frac{\lambda\rho_o}{\Lambda} + \frac{\beta-\rho_o}{\Lambda} \cdot \frac{\lambda\rho_o}{\beta-\rho_o} - \frac{\lambda}{\Lambda}(\beta-\rho_o) - \frac{\lambda\rho_o}{\Lambda} = -\frac{\lambda\beta}{\Lambda} . \quad (1.3.28)$$

So Equation (1.3.23) becomes, using Equation (1.3.28),

$$n(t) = n(o) \left[\frac{\beta}{\beta-\rho_o} e^{\frac{\lambda\rho_o}{\beta-\rho_o} t} - \frac{\rho_o}{\beta-\rho_o} e^{-\frac{\beta-\rho_o}{\Lambda} t} \right] . \quad (1.3.29)$$

This approximate solution is very useful in obtaining checks on numerical solutions. The character of the solution is also exponential. This will play a role in our numerical technique.

Problem 1.3.1

Assume that $\rho(t) = 0$ in Equations (1.3.3) and (1.3.4) and obtain a solution of the kinetics equations.

Problem 1.3.2

Prove the theorems

a) $|\underline{B}^{-1} \underline{A} \underline{B} - \omega \underline{I}| = |\underline{A} - \omega \underline{I}|$

and

b) $\text{trace } \underline{A} = \sum_{i=1}^{2} \omega_i$

Problem 1.3.3

Why does Equation (1.3.17) follow from the eigenvector equation

$\underline{A} \underline{v} = \omega \underline{v}$?

Problem 1.3.4

Using the same approximations as used in developing Equation (1.3.29) establish a relation for c(t).

1.4 Numerical Solution of the Kinetics Equations

The reactor kinetics equations are difficult to solve numerically by "standard" Runge-Kutta of predictor-corrector methods. The basic reason becomes apparent by looking at the one-delayed neutron group equations which we repeat as

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda}\, n(t) + \lambda\, c(t) \qquad (1.4.1)$$

and

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda}\, n(t) - \lambda\, c(t)\ . \qquad (1.4.2)$$

The very short time response Λ of the prompt neutrons is of the order of 10^{-4} sec. whereas the delayed neutron time response is $\frac{1}{\lambda}$ or about 10 sec, a factor of 10^5 greater. The implication of these facts is that in order to obtain the prompt response, very small time steps, of the order of 10^{-4}sec, are required. But then before the delayed neutron term can come into play, many time steps are required. Also, to examine the response out to even one second, 10,000 steps of calculation would be required.

There are several methods that are used to ameliorate this difficult problem:

1) Using a Laplace transform technique.

2) Transferring the differential equations to integral equations.

3) Using the eigenvalue method.

We choose the last technique and refer to it as Hansen's method (4) after its originator. The method works for varying reactivity and can be extended

to systems with feedback.

The basic idea of Hansen's method is relatively simple. We again write Equations (1.4.1) and (1.4.2) as a matrix and set

$$\frac{d\underline{\psi}(t)}{dt} = \underline{\underline{A}}\,\underline{\psi} \tag{1.4.3}$$

where A and $\underline{\psi}$ are defined as in Section 3 of this module. We will also only perform the operations for our one delayed neutron group model. Now set

$$\underline{\underline{A}} = \underline{\underline{L}} + \underline{\underline{D}} + \underline{\underline{U}} \tag{1.4.4}$$

where

$$\underline{\underline{L}} = \begin{bmatrix} 0 & 0 \\ \frac{\beta}{\Lambda} & 0 \end{bmatrix}, \quad \underline{\underline{D}} = \begin{bmatrix} \frac{\rho-\beta}{\Lambda} & 0 \\ 0 & -\lambda \end{bmatrix}$$

and

$$\underline{\underline{U}} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix}.$$

Of course, for all six delayed neutron groups these matrices would still have the same meaning. Equation (1.4.4) can be written using these definitions as

$$\frac{d\underline{\psi}(t)}{dt} - \underline{\underline{D}}\,\underline{\psi}(t) = (\underline{\underline{L}} + \underline{\underline{U}})\,\underline{\psi}(t) . \tag{1.4.5}$$

Equation (1.4.5) does not appear any simpler to solve than Equation (1.4.3) and in fact it isn't. The reason for splitting it up in this fashion is to develop an iteration procedure. We assume we begin this calculation from a time t_0 and advance to a time t_1. We set

$$h = t_1 - t_0 \,. \tag{1.4.6}$$

Since $\underline{D}$ is a diagonal matrix, an integrating factor for Equation (1.4.5) is $e^{-\underline{D}t}$ if the reactivity doesn't change much during the time interval h. Therefore, Equation (1.4.5) becomes

$$e^{-\underline{D}t} \frac{d\underline{\psi}(t)}{dt} - e^{-\underline{D}t} \underline{D}\, \underline{\psi}(t) = e^{-\underline{D}t} (\underline{L} + \underline{U})\, \underline{\psi}(t)$$

or

$$\frac{d}{dt} \left(e^{-\underline{D}t} \underline{\psi}(t)\right) = e^{-\underline{D}t} (\underline{L} + \underline{U})\, \underline{\psi}(t) \tag{1.4.6}$$

Integrating now from 0 to h we have

$$e^{-\underline{D}t} \psi(t) \Big|_0^h = \int_0^h e^{-\underline{D}t} (\underline{L} + \underline{U})\, \underline{\psi}(t)\, dt$$

or

$$\psi(t_o + h) = e^{\underline{D}h} \underline{\psi}(t_o) + \int_0^h e^{\underline{D}(h-\theta)} (\underline{L} + \underline{U})\, \underline{\psi}(t_o + \theta) d, \tag{1.4.7}$$

where

$$t_o \leq \theta \leq t_1 = t_o + h \,,$$

and obviously in this interval

$$d\theta = dt.$$

This is an integral equation since the function we're looking for is part of the integrand. In order to provide a reasonable approximation to $\underline{\psi}(t_o + \theta)$, we recall that the analytical solution is exponential so we assume

$$\underline{\psi}(t_o + \theta) = e^{\omega_o \theta}\, \underline{\psi}(t_o) , \qquad (1.4.8)$$

and ω_o is the largest eigenvalue of the matrix $\underline{\underline{A}}$. This means that we will have to solve the equation (notice this is simply the inhour equation)

$$|\underline{\underline{A}} - \omega \underline{\underline{I}}| = 0 \qquad (1.4.9)$$

in each time interval for which the reactivity has changed since the reactivity will generally be a time dependent quantity.

Inserting Equation (1.4.8) into (1.4.7) we have

$$\underline{\psi}(t_o + h) = e^{\underline{\underline{D}}h}\, \underline{\psi}(t_o) + \int_o^h e^{\underline{\underline{D}}(h-\theta)} (\underline{\underline{L}} + \underline{\underline{U}})\, e^{\omega_o \theta}\, \underline{\psi}(t_o)\, d\theta$$

$$= e^{\underline{\underline{D}}h}\, \underline{\psi}(t_o) + \left[\int_o^h e^{\underline{\underline{D}}(h-\theta)}\, e^{\omega_o \underline{\underline{I}}\theta}\, d\theta \right] (\underline{\underline{L}}+\underline{\underline{U}})\, \underline{\psi}(t_o)$$

$$= e^{\underline{\underline{D}}h}\, \underline{\psi}(t_o) + \left[e^{\underline{\underline{D}}h} \cdot \int_o^h e^{(\omega_o \underline{\underline{I}} - \underline{\underline{D}})\theta}\, d\theta \right] (\underline{\underline{L}}+\underline{\underline{U}})\, \underline{\psi}(t_o)$$

$$= e^{\underline{\underline{D}}}\, \underline{\psi}(t_o) + (\omega_o \underline{\underline{I}} - \underline{\underline{D}})^{-1} \left[e^{\omega_o h \underline{\underline{I}}} - e^{\underline{\underline{D}}h} \right] (\underline{\underline{L}}+\underline{\underline{U}})\, \underline{\psi}(t_o) . \qquad (1.4.10)$$

If we write

$$\psi(t_o) = \psi_j \tag{1.4.11}$$

and

$$\psi(t_o + h) = \psi_{j+1}$$

Then Equation (1.4.10) becomes

$$\underline{\psi}_{j+1} = e^{\underline{\underline{D}}h}\,\underline{\psi}_j + (\omega_o\underline{\underline{I}} - \underline{\underline{D}})^{-1}\left[e^{\omega_o h\underline{\underline{I}}} - e^{\underline{\underline{D}}h}\right](\underline{\underline{L}} + \underline{\underline{U}})\underline{\psi}_j \,. \tag{1.4.11}$$

$$\equiv \underline{\underline{G}}\,\underline{\psi}_j. \tag{1.4.12}$$

This $\underline{\underline{G}}$ matrix obviously represents

$$\underline{\underline{G}} = e^{\underline{\underline{D}}h} + (\omega_o\underline{\underline{I}} - \underline{\underline{D}})^{-1}\left[e^{\omega_o\underline{\underline{I}}h} - e^{\underline{\underline{D}}h}\right](\underline{\underline{L}} + \underline{\underline{U}}), \tag{1.4.13}$$

and it can be written as

$$\underline{\underline{G}} = \begin{bmatrix} e^{\frac{\rho-\beta}{\Lambda}h} & \dfrac{e^{\omega_o h} - e^{\frac{\rho-\beta}{\Lambda}h}}{\omega_o - \left(\frac{\rho-\beta}{\Lambda}\right)}\,\lambda \\ \dfrac{e^{\omega_o h} - e^{-\lambda h}}{\omega_o + \lambda_1}\,\dfrac{\beta}{\Lambda} & e^{-\lambda h} \end{bmatrix} \tag{1.4.14}$$

For completeness, if there are N delayed neutron groups, we include the following relation for $\underline{\underline{G}}$,

$$\underline{G} \begin{bmatrix} e^{\frac{\rho-\beta}{\Lambda}h} & \frac{e^{\omega_o h} - e^{\frac{\rho-\beta}{\Lambda}h}}{\omega_o - (\frac{\rho-\beta}{\Lambda})}\lambda_1 & \cdots & \frac{e^{\omega_o h} - e^{\frac{\rho-\beta}{\Lambda}h}}{\omega_o - (\frac{\rho-\beta}{\Lambda})}\lambda_N \\ \frac{e^{\omega_o h} - e^{-\lambda_1 h}}{\omega_o + \lambda_1}\frac{\beta_1}{\Lambda} & e^{-\lambda_1 h} & \cdots & 0 \\ \vdots & & & \vdots \\ \frac{e^{\omega_o h} - e^{-\lambda_N h}}{\omega_o + \lambda_N}\frac{\beta_N}{\Lambda} & \cdots\cdots\cdots & & e^{-\lambda_N h} \end{bmatrix} \quad (1.4.15)$$

If there are six delayed neutron groups, it is apparent that $\underline{G}$ will be a 7 x 7 matrix. Also this iteration technique as given by Equation (1.4.12) is unconditionally stable and yields the asymptotically correct eigen-solution.

The numerical procedure for the solution of the kinetics equations is thus:

1. Determine the number of delayed neutron groups desired and read in the pertinent parameters such as Λ_1, β_1 etc. Also choose a time step "h".
2. Construct the vector $\underline{\psi}(0)$. This will usually be

$$\underline{\psi}(0) = n(0) \begin{bmatrix} 1 \\ \beta_1/\lambda_1\Lambda \\ \vdots \\ \beta_N/\lambda_N\Lambda \end{bmatrix} .$$

3. Determine the largest eigenvalue of the equation

$$|\underline{A} - \omega\underline{I}| = 0.$$

This is a rather difficult step since it means solving an algebraic equation of perhaps degree 7 to determine its largest root ω_o.

4. Construct the $\underline{\underline{G}}$ matrix using Equation (1.4.15).
5. Determine the vector $\underline{\psi}_1$ where

$$\underline{\psi}_1 (h) = G\, \underline{\psi}(0).$$

6. Repeat the above steps starting with step 3.

The technique is not involved and can yield very accurate results.

The determination of the root of an algebraic equation needs some discussion. We choose the Newton-Raphson technique to solve the equation

$$f(\omega) = \sum_{n=0}^{N} a_n \omega^n = 0 \quad \text{for} \quad 1 \leq N \leq 7. \tag{1.4.16}$$

Let the N roots of Equation (1.4.17) be labelled $\omega_o, \omega_1, \ldots, \omega_N$ where the roots are ordered such that $\omega_o > \omega_1 > \omega_2 \ldots > \omega_7$. For our problem, all the roots are real and there will only be one positive root depending on whether ρ is positive at the time step of interest. We are interested only in ω_o. Also we assume that the interval of interest in the roots is limited by

$$\omega_o \leq \left|\frac{\rho_o}{\Lambda}\right|$$

and

$$|\omega_N| \leq |\lambda_N|,$$

with λ_N the decay constant corresponding to the shortest lived delayed neutron group. The reactivity can range from negative β to positve β in

most practical situations. Note that if

$$\rho = 0 \text{ then } \omega_o = 0$$

and if $\rho = \beta$, then $\omega_o \to \infty$ while if $\rho = -\beta$, then $\omega_o \to -\lambda_1$. For the last situation, regardless of the amount of negative reactivity introduced into the reactor, the reactor cannot shut down faster than a period of

$$T = \omega_o^{-1} = \frac{1}{\lambda_1} \simeq 80 \text{ sec.}$$

The Newton-Raphson method is relatively simple to use. Assume that we can expand $f(\omega)$ in a Taylor series about ω_o, where ω_o is the root of interest. Then

$$f(\omega_o) = f(\omega_{o_1}) + h \frac{df(\omega_{o_1})}{d\omega} + \frac{h^2}{2} \frac{d^2 f(\omega_{o_1})}{d\omega^2} + \ldots \quad (1.4.17)$$

The ω_{o_1} is a first "guess" at the solution which we assume to be $\rho(t)/\Lambda$. If we truncate Equation (1.4.17) after the first two terms on the right, we have

$$f(\omega_o) \simeq f(\omega_{o_1}) + h \frac{df}{d\omega}(\omega_{o_1}) = 0$$

or

$$h = \frac{-f(\omega_{o_1})}{\frac{df(\omega_{o_1})}{d\omega}} \quad (1.4.18)$$

The next approximation to ω_o is then

$$\omega_{o_2} = \omega_{o_1} + h \quad (1.4.19)$$

and then Equation (1.4.18) is used again. If this iteration procedure is used a sufficient number of times, then the roots of Equation (1.4.16) can be obtained.

Problem 1.4.1

Use the Newton-Raphson method to solve the equation

$$x^2 + 3x - 8 = 0.$$

Check by the quadratic formula.

Problem 1.4.2

Extend the Newton-Raphson method to systems of equations. In particular if

$$f_1 (x_1, x_2) = 0$$

and

$$f_2 (x_1, x_2) = 0$$

then show that if h_1 and h_2 are the increments to the assumed roots,

$$h_1 = \frac{f_2 \frac{\partial f_1}{\partial x_2} - f_1 \frac{\partial f_2}{\partial x_2}}{\det \underline{J}}$$

and

$$h_2 = \frac{f_1 \frac{\partial f_2}{\partial x_2} - f_2 \frac{\partial f_1}{\partial x_1}}{\det \underline{J}}$$

where $\underline{J}$ is the Jacobian matrix, i.e.

$$\underline{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

and det $\underline{J}$ is the determinant of the Jacobian matrix.

Using this formulation find the solution to the equations

$$f_1(x_1, x_2) = \sin x_1 x_2 - x_1 + x_2^2 = 0$$

$$f(x_1, x_2) = 2x_1^2 - x_2^2 + x_1 x_2 = 0.$$

1.5 The Computer Program and the Kinetics Equations

As has already been indicated, the kinetics equations offer a challenge for their successful computer solution. The purpose of this section is to indicate what models the program will solve. The program will solve the kinetics equations by means of Hansen's method for the following reactivity inputs:

1) A constant reactivity,

$$\rho(t) = \rho_o. \tag{1.5.1}$$

2) Sinsusoidal variation of reactivity,

$$\rho(t) = \rho_o \sin b_2 t, \tag{1.5.2}$$

where ρ_o and b_2 must be input.

3) Linear reactivity insertion to approximate the insertion or withdrawal of a control rod:

$$\rho(t) = \rho_o(1 + b_3 t). \tag{1.5.3}$$

These inputs are the most common and can be used to simulate many situations. In each of the above situations ρ_o must be read into the computer as well as the "b" or rate of insertion in sec^{-1}. The reactivity units are in dollars. A dollar of reactivity is the amount inserted or withdrawn which equals the delayed neutron fraction. For example, a reactivity of 50¢ for U-235 where $\beta = 0.0065$ is 0.00325 whereas for Pu-239, it is 0.00135 since $\beta = 0.0027$.

The program also has an option of either doing the calculations for

U-235 or Pu-239. The delayed neutron decay constants are taken from Keepins' data and are given in Table 1.5.1.

Table 1.5.1 Delayed-Neutron Half-Lives, Decay Constants and Yields from Thermal Fission of U-235 and Pu-239.

	URANIUM-235			PLUTONIUM-239		
Group Index	Half-Life (sec)	Decay Constant λ (sec^{-1})	Relative Abundance $\beta i/\beta$	Half-Life (sec)	Decay Constant λ (sec^{-1})	Relative Abundance $\beta i/\beta$
1	55.72	0.0124	0.033	54.28	0.0128	0.035
2	22.72	0.0305	0.219	23.04	0.0301	0.298
3	6.22	0.111	0.196	5.60	0.124	0.211
4	2.30	0.301	0.395	2.13	0.325	0.326
5	0.610	1.14	0.115	0.618	1.12	0.086
6	0.230	3.01	0.042	0.257	2.69	0.044
		β=0.0065			β=0.0027	

The computer program for this module is good only for low power reactors since feedback effects are not taken into account. It does give some idea of the behavior of the power of the reactor for various reactivities as well as for the two different fuels.

1.6 Input-Output Data for Code FUMOKI

The input data required for the program are presented below:

Card 1. IFUEL indicates the type of fuel the reactor has. It can either be ^{235}U or ^{239}Pu. If IFUEL = 5 - its Uranium -235

IFUEL = 9 - the fuel is Plutonium 239.

The format is I2.

Card 2. NN - The number of groups of delayed neutrons desired. NN can be any integer from 1 to 6.

The format is again I2.

Card 3. NRO - Type of reactivity inserted into the reactor.

The format is I2.

If NRO = 1, $\rho(t) = \rho_o$,

NRO = 2, $\rho(t) = \rho_o \sin b_2 t$,

NRO = 3, $\rho(t) = \rho_o(1 + b_3 t)$.

Card 3 must have the reactivity ρ_o, in dollars, read in with an F 12.9 format. The b_2 and b_3 are read in with units of sec^{-1}. The b's are read with an F 12.9 format on this card.

Card 4. XL - Neutron generation time.

XL must be in seconds and is in an F 12.9 format.

Card 5. TIME: - the total time period for which the solution is desired (seconds)

H - The time step desired, again in seconds. This cannot be taken arbitrarily large. Generally one should choose an H of between 0.005 and 0.05 sec. Both of these are F 10.5 formats.

Card 6. IG - Output indicator. This indicates whtther the user would like to print out the G matrix, the flux and time step or not. If IG = 1, then the flux, "G" matrix and time step are all printed. If IG = 2, G is not printed out. This is an I2 format.

Card 7. XN - The initial neutron density.

XN can be anything you choose but it is in an F 10.5 format.

An example of an input data set is given below. This is an example where we use Uranium fuel, with one group of delayed neutrons for a constant reactivity input of about 34¢.

The core requirements for FUMOKI are about 40 K bytes and the compilation time increases approximately with the square of the number of delayed neutron groups, all other parameters being the same.

The output of the program is simple. The time, neutron density and G matrix can be read out. Also a plot of the density versus time is printed out.

Problem 1.6.1

Run the sample problem, i.e. the data shown.

Problem 1.6.2

Find the neutron density if you start with 100 neutrons/cm^3 for a ^{239}Pu reactor with all six delayed neutron groups. Obtain the solution for a 5 second time span and do the same problem with H = 0.01 and 0.1 sec. Compare the results. Do not write the $\underline{\underline{G}}$ matrix.

Problem 1.6.3

Run FUMOKI for the case for three delayed neutron groups for ^{239}Pu, with a neutron generation time of 10^{-4} sec and the reactivity

$$\rho(t) = \$0.25 \sin 5t,$$

Run it for a total time of 1 second, time intervals of 0.01 sec and an initial relative flux of 1.000. Notice that the reactivity at 0.5 sec is \$0.17 and the power has risen to 1.275.

If you do the exact same problem as above for 2 delayed neutron groups notice that the power has become 1.268 at 0.5 sec.

Problem 1.6.4

Given a reactor fueled with ^{239}Pu and assume that you wish to use a 5 delayed-neutron-group model. For a relative power of 1.000 initially, show that after one second the power is 2.157 if a rod is being removed such that the reactivity inserted is

$$\rho(t) = \rho_o(1 + 5t).$$

All other parameters are as in problem 1.6.3.

Problem 1.6.5

Assume a ^{235}U fueled system with a reactivity variation of

$$\rho(t) = \$0.25 \sin 5t.$$

If the reactor is to be simulated for three seconds after the reactivity is inserted and if all parameters are the same as in example 1.6.3 except for the number of delayed neutron groups then:

a) use the two delayed neutron model to obtain the power as a function of time

and

b) use the five delayed neutron model to obtain the power variation with time.

How much longer did the computer take to do case b) than case a)?

Answer: About 10 times as long. On the IBM 370, the times were 11.03 sec and .97 sec respectively. Notice also that the power at the end of 3.0 sec is 0.8154 and 0.8161 for the 2 and 5 group cases respectively.

REFERENCES

1. G. I. Keepin, "Physics of Nuclear Kinetics", Addison-Wesley Publishing Co., Inc., (1965) pages 73-129.

2. D. L. Hetrick, "Dynamics of Nuclear Reactors", The University of Chicago Press, (1971) pages 111-139.

3. P. F. Zweifel, "Reactor Physics", McGraw-Hill Book Company, (1973) pages 78-100.

4. K. F. Hansen, B. V. Koen and W. W. Little, Jr. Nuclear Science and Engineering 22, (1965) pages 51-59.

List of Symbols for FUMOKI

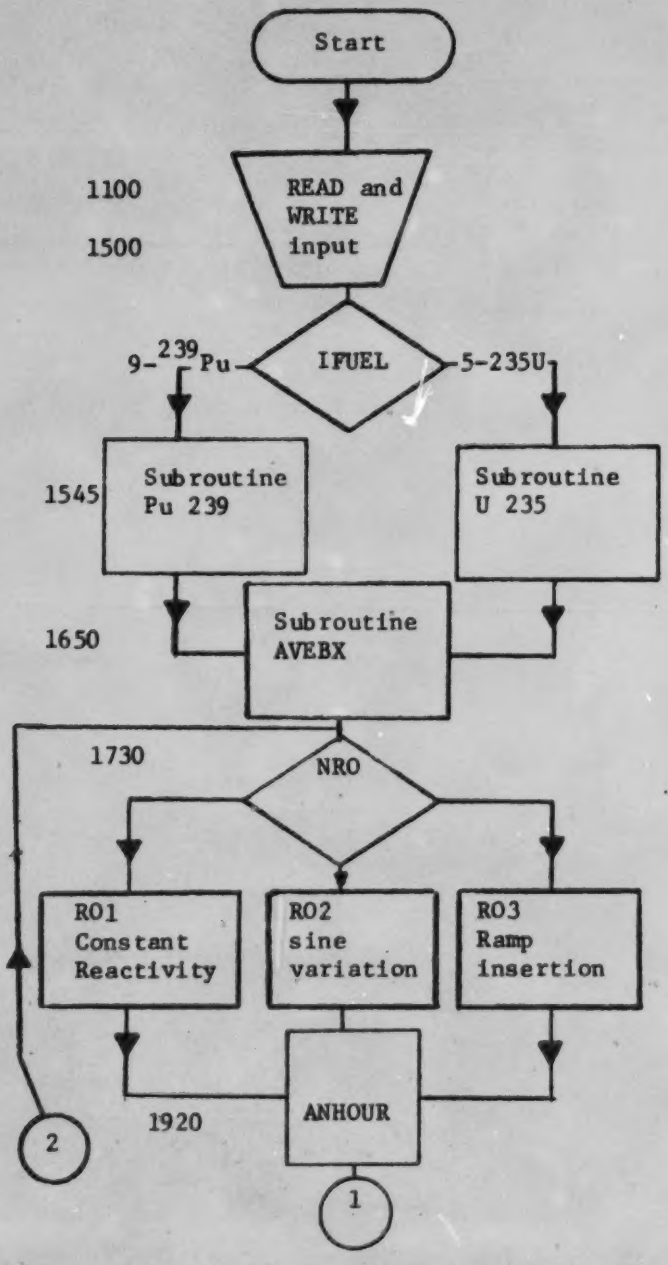
The following symbols are listed in the order of their appearance in program FUMOKI.

IFUEL		Type of fuel (^{235}U or ^{239}Pu) (5 or 9)
NN		Number of delayed neutron groups
NRO	$\rho(t) = \rho_o$ $\rho_o \sin b_2 t$ $\rho_o(1 + b_3 t)$	Type of reactivity to be inserted. 1 = constant reactivity 2 = sine insertion 3 = ramp insertion
RO	ρ_o	Reactivity inserted
B1,B2,B3	b_1, b_2, b_3	Rate or period at which reactivity is inserted
X(I)	λ	precursor decay constants
XL	Λ	neutron generation time
TIME		total time the reactor transient (s) is to be simulated
H	Δt	the time step
IG		output option to print the $\underline{G}$ matrix after 1st iteration(to print any other iteration change statement MKI 2520)
XN	n_o	initial relative power
B(I)		fraction of neutrons for each group
BB		total delayed neutron fraction
TH		cumulative time used in TPLOT
LL		total number of time steps H fits into TIME
R	$\rho(t)$	Reactivity at any time
MM		number of coefficients in the inhour equation. (number of delayed neutron groups + 2)

AW(I)		coefficients of the inhour polynomial
M		degree of the inhour polynomial (number of delayed neutron groups + 1)
WO	ω_o	largest eigenvalue of $\lvert\underline{A} - \omega\underline{I}\rvert = 0$
XNCO(I), GNXO(I)	$\underline{\phi}$	column vector $\underline{\phi}$
XXNC2		logarithm of power
GG,MAD		power
MOD(I)	H*(step number)	dimensioned TH used in TPLOT
Z1(I)		precursor density of delayed neutron group 1
Z2(I)		reactivity
AVEBX		subroutine to obtain the average β and λ depending on how many delayed neutron groups are desired
ANHOUR		subroutine to calculate the coefficients of the inhour equation
GMTRX		a subroutine which forms the $\underline{G}$ matrix
GXN		subroutine which multiplies $\underline{G}$ by column vector $\underline{\phi}$, i.e. $G\underline{\phi}_o$
POLRT		A Newton-Raphson iteration technique is used to obtain the roots of the inhour equation to obtain ω_o
TPLOT		A subroutine to plot the vector $\underline{\phi}$ up to and including five dependent variables. The reactivity, logarithm of the power, power and some of the delayed neutron precursors are plotted.

Flow Chart for FUMOKI

Statement Numbers



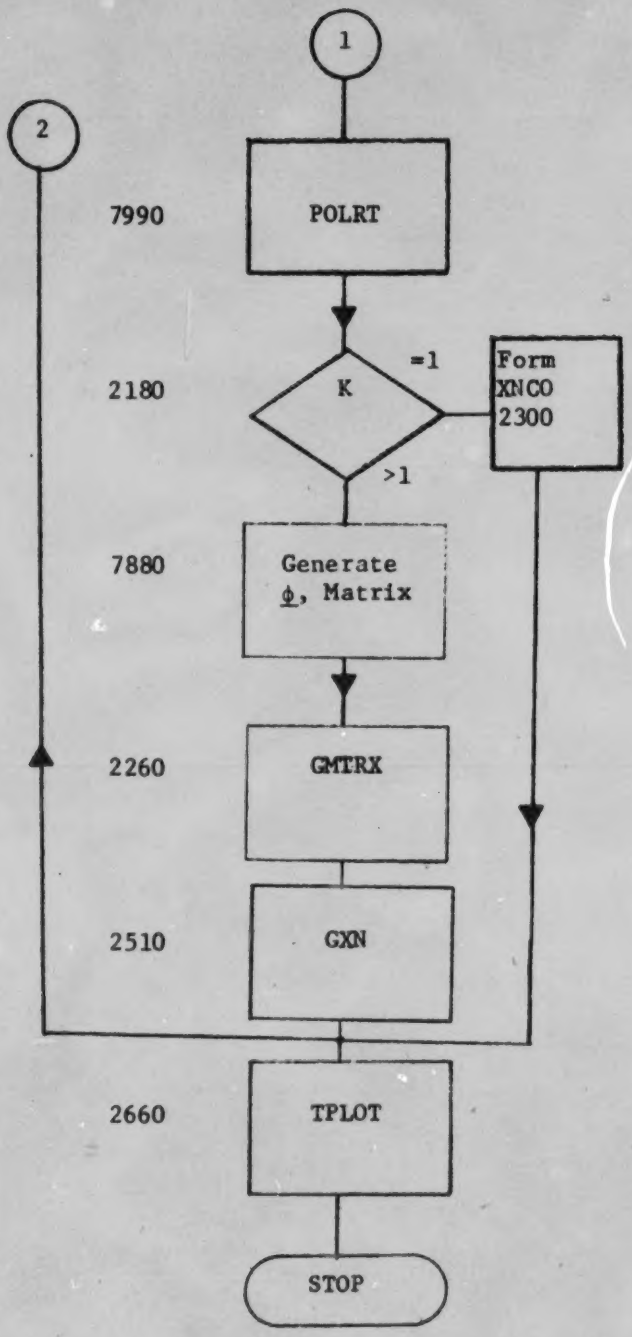
Obtains six group β's and λ's for isotope.

Calculates average β's and λ's depending on the number of delayed groups desired.

The type of reactivity variation desired determines which path to take.

Routines which determine the reactivity as functions of time.

Subroutine which calculates the polynomial form of the inhour equation.



Subroutine to determine the largest root of the inhour equation using Newton-Raphson method.

Subroutine which forms initial $\underline{\phi}$ matrix; $\underline{\phi}_o$

Generates the $\underline{\phi}$ matrix for all cases except the first

Forms the $\underline{\underline{G}}$ matrix

Forms the product $\underline{\underline{G}}\ \underline{\phi}$

Plots the power, etc.

```
//WATFIV SMART,ONEGA,PAGES=80,TIME=150
C      MAIN PROGRAM                                                    MK1   20
C                                                                      MK1   30
C                                                                      MK1   40
C                              KINETICS MODULE                         MK1   50
C                                                                      MK1   60
C                                                                      MK1   70
C     CODE NAME      F U N K I  .                                      MK1   80
C     OBJECTIVE    TO OBTAIN THE KINETICS EQUATION WITHOUT FEEDBACK AND MK1   90
C                SOLVE THE KINETICS EQUATIONS NUMERICALLY FOR ONE , TO MK1  100
C                SIX DELAYED NEUTRON GROUPS FOR TIME VARYING REACTIVITY MK1  110
C                INSERTIONS .                                          MK1  120
C                THE KINETIC EQUATION DERIVED FROM ONE GROUP DIFFUSION MK1  130
C                EQUATIONS .                                           MK1  140
C                FEEDBACK EFFECTS AND SPATIAL DEPENDENT EFFECTS ARE NOT MK1  150
C                INCLUDED .                                            MK1  160
C                                                                      MK1  170
C     ASSUMPTION  THE BASIC ASSUMPTION OF THE METHOD IS THAT NEUTRON ANDMK1  180
C                PRECURSOR DENSITIES BEHAVE EXPONENTIALLY WITH A FREQUENCYMK1  190
C                CHARACTERISTICS OF THE ASYMPTOTIC FREQUENCY CORRESPON- MK1  200
C                DING TO THE REACTIVITY .                              MK1  210
C                                                                      MK1  220
C     PROGRAM      WRITTEN IN SINGLE PRECISION                         MK1  230
C                                                                      MK1  240
C                GENERAL DESCRIPTION OF PARAMETERS                     MK1  250
C                                                                      MK1  260
C     SYMBOL  IN/OUT/V   DESCRIPTION              REAL/INT.   UNIT     MK1  270
C                                                                      MK1  280
C     IFUEL     IN         TYPE OF FUEL             I          -       MK1  290
C                        5 = U-235                                     MK1  300
C                        9 = PU-239                                    MK1  310
C     NN        IN         GROUP DELAYED NEUTRON    I          -       MK1  320
C                        FROM 1 TO 6 .                                 MK1  330
C     NRC       IN         TYPE OF REACTIVITY       I          -       MK1  340
C                        1= CONSTANT                                   MK1  350
C                        2= FSIN(B2*T)                                 MK1  360
C                        3= R(1+P3*T)                                  MK1  370
C     B1        IN         CONSTANT B2 OR B3        R          1/SEC   MK1  380
C                        IN  ARO                                       MK1  390
C     B         OUT        FRACTION OF DELAYED      R          -       MK1  400
C                        NEUTRON                                       MK1  410
C     X         OUT        PRECURSOR DECAY          R          1/SEC . MK1  420
C                        CONSTANT                                      MK1  430
C     XL        IN         NEUTRON GENERATION       R          SEC .   MK1  440
C                        TIME                                          MK1  450
C     R         V          REACTIVITY               R          -       MK1  460
C     A         V          COEFFICIENT OF INHOUR    R          -       MK1  470
C                        FORMULA IN POLYNOMIAL                         MK1  480
C                        FORM                                          MK1  490
C     WO        V          RECIPROCAL OF PERIOD     R          1/SEC   MK1  500
C     T         V          STABLE PERIOD            R          SEC .   MK1  510
C     G         V          G - MATRIX               R          -       MK1  520
C     XN        V          NEUTRON DENSITY          R          N/CM**3 MK1  530
C     C         V          PRECURSOR DENSITY        R          N/CM**3 MK1  540
C     TIME      N          TOTAL TIME               R          SEC     MK1  550
C     H         N          TIME INCREMENT           R          SEC     MK1  560
C                                                                      MK1  570
C                                                                      MK1  580
C     IN      - INPUT                                                  MK1  590
C     OUT     - OUTPUT                                                 MK1  600
```

```
C     V       - VARIABLES                                                    MK1  610
C                                                                            MK1  620
C                                                                            MK1  630
C     --------------THIS PROGRAM  IS LIMITED TO DELAYED NEUTRONS FROM THMK1  640
C           THERMAL FISSION OF U-235 AND PU-239 FOR ANY NUMBER OF          MK1  650
C           GROUPS FROM ONE TO SIX .                                       MK1  660
C                                                                          MK1  670
C           SUPPORTING ROUTINE    U235,PU239,AVEBX                         MK1  680
C                                 R01,R02, AND R03 .                       MK1  690
C                                                                          MK1  700
C                                                                          MK1  710
      1     REAL MAD,MOD                                                   MK1  720
      2     DIMENSION Z1(100), Z2(100), GG(100), MAD(100), MOD(100), XXNC2(100MK1  730
           1), HH(100)                                                     MK1  740
      3     DIMENSION B(6), XX(6), BX(6), X(6), YB(6), YX(6), A(8), DER1(8), FMK1  750
           1W1(8), G(4,4), AW(8), ROUT1(7), COF(7), W(7), XNCO(7), GNXCO(7), CMK1  760
           2(6)                                                            MK1  770
C                                                                          MK1  780
C     --------------DEFINE REACTIVITY AS A FUNCTION OF TIME .              MK1  790
C                                                                          MK1  800
      4     RO2(RO,T,B2)=RO*SIN(B2*T)                                      MK1  810
      5     RO3(RO,T,B3)=RO*(1.0+B3*T)                                     MK1  820
C                                                                          MK1  830
C                                                                          MK1  840
C                          INPUT PARAMETER DATA                            MK1  850
C                                                                          MK1  860
C                                                                          MK1  870
C     110       IFUEL  EITHER 5 OR 9 .                                     MK1  880
C                      5 - FUELED U-235                                    MK1  890
C                      9 - FUELED U-239                                    MK1  900
C     120       NN      GROUP DELAYED NEUTRON                            MK1  910
C     130       NRO     TYPE OF REACTIVITY                               MK1  920
C                      1 - CONSTANT REACTIVITY                             MK1  930
C                      2 - REACTIVITY AS A FUNCTION OF TIME .              MK1  940
C                        R = RO*SIN(B2*T)                                  MK1  950
C                      3 - REACTIVITY AS A LINEAR FUNCTION OF T .          MK1  960
C                        R = RO*(1+B3*T)                                   MK1  970
C                RO     CONSTANT / INITIAL REACTIVITY .                   MK1  980
C                B1     EITHER B2 OR B3 IN NRO                            MK1  990
C     140        AL      NEUTRON GENERATION TIME                          MK1 1000
C     150        TIME    TOTAL TIME USED                                  MK1 1010
C                H       TIME INTEAVAL                                    MK1 1020
C     160        IG      OUTPUT SIGNAL FOR G-MATRIX                       MK1 1030
C                          1 - PRINT OUT G-MATRIX                         MK1 1040
C                          2 - DON'T .                                    MK1 1050
C     170        XN      INITIAL RELATIVE FLUX                            MK1 1060
C                                                                          MK1 1070
C     ****INPUT REACTIVITY IN DOLLAR UNIT *******                          MK1 1080
C                                                                          MK1 1090
C                                                                          MK1 1100
C     --------------READ THE INPUT DATA, STEP 1 P. 26                      MK1 1110
      6     READ (5,110) IFUEL                                             MK1 1120
      7     READ (5,120) NN                                                MK1 1130
      8     READ (5,130) NRO,RO,B1                                         MK1 1140
      9     READ (5,140) XL                                                MK1 1150
     10     READ (5,150) TIME,H                                            MK1 1160
     11     READ (5,160) IG                                                MK1 1170
     12     READ (5,170) XN                                                MK1 1180
     13     WRITE (6,20)                                                   MK1 1190
     14     WRITE (6,30) IFUEL,NN                                          MK1 1200
```

```
15          WRITE (6,40) XL                                                   MK1 1210
16          WRITE (6,50) NKC,RC,NPG,M1                                        MK1 1220
17          WRITE (6,60) TIME,H                                               MK1 1230
18          WRITE (6,70) IC,XN                                                MK1 1240
19          WRITE (6,100)                                                     MK1 1250
20    20    FORMAT (1H1,69H********************************* KINETICS MODULE 1 **MK1 1260
           1*********** ****,///,20H**********,/,1X,16H INPUT PARAMK1 1270
           2METER,/,20H**********,//////)                                    MK1 1280
21    30    FORMAT (2X,18H TYPE OF FUEL U-23,I1,//,2X,29H NUMBER OF DELAYED NEMK1 1290
           1UTRON = ,I2,/)                                                   MK1 1300
22    40    FORMAT (2X,27H NEUTRON GENERATION TIME = ,F11.5,2X,'(SEC)',/)     MK1 1310
23    50    FORMAT (2X,22H TYPE OF REACTIVITY = ,I2,//,2X,10H RO =  $  ,F15.3,MK1 1320
           15X,'R',I1,' = ',F11.5,/)                                         MK1 1330
24    60    FORMAT (2X,19H TOTAL TIME USED = ,F10.3,2X,'(SEC)',5X,'  ,  TIME IMK1 1340
           <NTERVAL = ',F7.3,'  (SEC)',/)                                    MK1 1350
25    70    FORMAT (2X,17H OUTPUT OPTION = ,I2,//,2X,25H INITIAL RELATIVE FLUXMK1 1360
           1 = ,F8.3,///)                                                    MK1 1370
26    80    FORMAT (5X,22H SIX GROUP LAMBDA ARE ,2X,6(F7.4,2X),//)            MK1 1380
27    90    FORMAT (5X,20H SIX GROUP BETA ARE ,2X,6(F7.5,2X),//)              MK1 1390
28    100   FORMAT (//////,39H********** END OF INPUT DATA **********,//////)  MK1 1400
29    110   FORMAT (I2)                                                       MK1 1410
30    120   FORMAT (I2)                                                       MK1 1420
31    130   FORMAT (I10,2(F10.5))                                             MK1 1430
32    140   FORMAT (F10.6)                                                    MK1 1440
33    150   FORMAT (2(F10.5))                                                 MK1 1450
34    160   FORMAT (I2)                                                       MK1 1460
35    170   FORMAT (F10.5)                                                    MK1 1470
      C                                                                       MK1 1480
      C     -------------COMPUTE # OF ITERATION AND TYPE OF FUEL USED          MK1 1490
      C                                                                       MK1 1500
36          M1=1                                                            MK1 1510
37          BB=0.0                                                          MK1 1520
38          TH=0.0                                                          MK1 1530
39          M1=M                                                            MK1 1540
40          LL=(TIME+M1/2.)/M1                                              MK1 1550
41          IF (IFUEL-5) 190,180,190                                        MK1 1560
42    180   CALL U235 (B,X)                                                 MK1 1570
43          GO TO 200                                                       MK1 1580
44    190   CALL PU239 (B,X)                                                MK1 1590
45    200   WRITE (6,80) (X(I),I=1,6)                                       MK1 1600
46          WRITE (6,90) (B(I),I=1,6)                                       MK1 1610
      C                                                                     MK1 1620
      C     -------------COMPUTE THE AVERAGE BETA AND LAMDA FOR THE PARTICULAMK1 1630
      C               FUEL .                                                MK1 1640
      C                                                                     MK1 1650
47          CALL AVERA (B,X,6,NN)                                           MK1 1660
48          WRITE (6,210)                                                   MK1 1670
49    210   FORMAT (/,5X,27H THE AVERAGE BETA AND LAMDA,/,5X,28H-------------MK1 1680
           1-------------,/)                                                 MK1 1690
50          DO 220 L=1,NN                                                   MK1 1700
51    220   WRITE (6,230) L,B(L),L,X(L)                                     MK1 1710
52    230   FORMAT (6X,2HB(,I2,3H)= ,F11.5,5X,7HLAMBDA(,I2,4H) = ,F11.5,/)  MK1 1720
53          DO 460 K=1,LL                                                   MK1 1730
54          GO TO (240,250,260), NK1                                        MK1 1740
      C                                                                     MK1 1750
      C     -------------SEE EQUATION 1.5.1                                 MK1 1760
      C                                                                     MK1 1770
55    240   K=K1                                                            MK1 1780
56          GO TO 270                                                       MK1 1790
      C                                                                     MK1 1800
```

```
      C     --------------SEE EQUATION 1.5.2                                  MK1 1810
      C                                                                       MK1 1820
57    250   R=RO2(RO,TH,B1)                                                   MK1 1830
58          GO TO 270                                                         MK1 1840
      C                                                                       MK1 1850
      C     --------------SEE EQUATION 1.5.3                                  MK1 1860
      C                                                                       MK1 1870
59    260   R=RO3(RO,TH,B1)                                                   MK1 1880
      C                                                                       MK1 1890
      C     --------------COMPUTE THE COEFFICIENT OF INHOUR FORMULA, SEE EQUATMK1 1900
      C               ON 1.3.13 WHERE NN=1                                    MK1 1910
      C                                                                       MK1 1920
60    270   CALL ANHOUR (XL,X,B,R,A,NN)                                       MK1 1930
61          MM=NN+2                                                           MK1 1940
62          IF (K.EQ.1) WRITE (6,280)                                         MK1 1950
63    280   FORMAT (/,4X,32H COEFFICIENTS OF INHOUR FORMULA ,/,4X,35H---------MK1 1960
           1-------------------------,/)                                      MK1 1970
64          DO 300 I=1,MM                                                     MK1 1980
65          MW=MM-I+1                                                         MK1 1990
66          AW(MW)=A(I)                                                       MK1 2000
67          IF (K.GT.1) GO TO 300                                             MK1 2010
68          WRITE (6,290) I,AW(MW)                                            MK1 2020
69    290   FORMAT (/,5X,2HA(,I2,5H) =  ,F11.4)                               MK1 2030
70    300   CONTINUE                                                          MK1 2040
71          M=NN+1                                                            MK1 2050
      C                                                                       MK1 2060
      C     --------------COMPUTE THE ROOT OF THE POLYNOMIAL WITH MM COEFFICIEMK1 2070
      C               AND DETERMINE THE LARGEST EIGEN VALUE WO IN STEP 3      MK1 2080
      C                                                                       MK1 2090
72          CALL POLRT (A,COF,M,W,ROOTI,IER,MM)                               MK1 2100
73          WO=W(1)                                                           MK1 2110
74          DO 310 J=1,M                                                      MK1 2120
75          VO=AMAX1(W(J),WO)                                                 MK1 2130
76    310   WO=VO                                                             MK1 2140
77    340   IF (K.NE.1) GO TO 370                                             MK1 2150
78          WRITE (6,350) WO                                                  MK1 2160
79    350   FORMAT (//,6X,' THE LARGEST EIGEN-VALUE  =  ',E11.4,//)           MK1 2170
      C                                                                       MK1 2180
      C     --------------FORM THE INITIAL VECTOR COLUMN PHI .                MK1 2190
      C               INITIALIZE NEUTRON DENSITY, T=0 AND PRECURSOR DENSITY.  MK1 2200
      C                                                                       MK1 2210
80          XNCO(1)=XN                                                        MK1 2220
81          DO 360 I=2,M                                                      MK1 2230
82          J1=I-1                                                            MK1 2240
83          BB=BB+B(J1)                                                       MK1 2250
84          C(J1)=B(J1)/(X(J1)-XL)                                            MK1 2260
85    360   XNCO(I)=C(J1)*XN                                                  MK1 2270
86    370   IF(K.GT.1)GG(K)=GNXCO(1)                                          MK1 2280
87          GG(K)=XNCO(1)                                                     MK1 2290
88          RR=R*RR                                                           MK1 2300
89          IF(K-1)380,400,380                                                MK1 2310
90    380   M=M1                                                              MK1 2320
91          DO 390 J1=1,M                                                     MK1 2330
92    390   XNCO(J1)=GNXCO(J1)                                                MK1 2340
93          GO TO 410                                                         MK1 2350
94    400   M=0.0                                                             MK1 2360
      C                                                                       MK1 2370
      C     --------------CONSTRUCT THE G MATRIX CORRESPONDING TO EQUATION    MK1 2380
      C               1.4.15 AND STEP 4 AND 5 PAGE 26 AND MULTIPLY THE G-MA - MK1 2390
      C               TRIX BY INITIAL VECTOR .                                MK1 2400
```

```
      C                                                                 MK1 2410
                                                                        MK1 2420
   95 410  CALL CMTRX (RR,B,X,XL,WO,M,H,G)                             MK1 2430
   96      IF (K.NE.2.AND.IG.NE.1) GO TO 430                           MK1 2440
   97      WRITE (6,480)                                               MK1 2450
   98      DO 420 I=1,M                                                MK1 2460
   99 420  WRITE (6,490) (G(I,II),II=1,M)                             MK1 2470
  100 430  CALL GXN (G,XNCO,N,GNXCO)                                   MK1 2480
  101      IF (K.EQ.1) WRITE (6,440) NN                                MK1 2490
  102 440  FORMAT (///,2X,2HNO,3X,4HTIME,6X,1HR,7X,5HPOWER,5X,12,1X,21HGROUP MK1 2500
           1DELAYED NEUTRON,/,1X,4H(  ),1X,5H(SEC),5X,3H($),5X,7H(  -  ),7X,10MK1 2510
           2H(RELATIVE),//)                                             MK1 2520
  103      WRITE (6,450) K,TH,R,(GNXCO(I),I=1,M)                       MK1 2530
  104 450  FORMAT (1X,I3,2X,F7.3,2X,F5.2,7(2X,E11.4))                   MK1 2540
  105      TH=TH+H                                                     MK1 2550
  106      MOD(K)=TH                                                   MK1 2560
  107      DG=GG(K)                                                    MK1 2570
  108      MAD(K)=DG                                                   MK1 2580
  109      Z1(K)=GNXCO(2)                                              MK1 2590
  110      Z2(K)=R                                                     MK1 2600
  111 460  XXNC2(K)=ALOG(DG)                                          MK1 2610
  112      WRITE (6,500)                                               MK1 2620
  113      CALL TPLOT (MOD,MAD,Z1,XXNC2,Z2,LL)                         MK1 2630
  114 480  FORMAT (/,6X,14H THE G-MATRIX ,/,6X,15H --------------,/)   MK1 2640
  115 490  FORMAT (2X,4(E11.4,2X),/)                                   MK1 2650
  116 500  FORMAT (///,5X,70H ********************************* END OF CALCULATION ***MK1 2660
           1*****************)                                         MK1 2670
  117      STOP                                                        MK1 2680
  118      END                                                         MK1 2690
      C                                                               MK1 2700
      C                                                               MK1 2710
      C  ----------SUBROUTINE AVEBX IS CALCULATING THE AVERAGE BETA ANDMK1 2720
      C            LAMBDA WITH THE BASIC OF SIX GROUP .               MK1 2730
      C                                                               MK1 2740
      C            SUPPORTING ROUTINE ,  ABETA                        MK1 2750
      C                                  ALAMDA                       MK1 2760
      C                                                               MK1 2770
      C

                                                                      MK1 2780
  119      SUBROUTINE AVEBX (B,X,N,NGROUP)                            MK1 2790
  120      DIMENSION B(6), X(6), BX(6), XX(6), YB(6), YX(6)           MK1 2800
  121      DO 10 K=1,N                                                MK1 2810
  122      XX(K)=X(K)                                                 MK1 2820
  123 10   BX(K)=B(K)                                                 MK1 2830
  124      GO TO (20,30,60,90,130,150), NGROUP                        MK1 2840
  125 20   YB(NGROUP)=ABETA(B,N)                                      MK1 2850
  126      YX(NGROUP)=ALAMDA(B,X,N)                                   MK1 2860
  127      GO TO 150                                                  MK1 2870
  128 30   N1=NGROUP+1                                                MK1 2880
  129      DO 50 J=1,NGROUP                                           MK1 2890
  130      YB(J)=ABETA(B,N1)                                          MK1 2900
  131      YX(J)=ALAMDA(B,X,N1)                                       MK1 2910
  132      DO 40 J1=1,N1                                              MK1 2920
  133      N2=J1+3                                                    MK1 2930
  134      X(J1)=XX(N2)                                               MK1 2940
  135 40   B(J1)=BX(N2)                                               MK1 2950
  136 50   CONTINUE                                                   MK1 2960
  137      GO TO 150                                                  MK1 2970
  138 60   N3=NGROUP-1                                                MK1 2980
  139      J3=2
```

```
      DO 80 J1=1,NGROUP                                                 MK1 2990
      YB(J1)=ABETA(B,N3)                                                MK1 3000
      YX(J1)=ALAMDA(B,X,N3)                                             MK1 3010
      IF (J1.EQ.NGROUP) GO TO 150                                       MK1 3020
      DO 70 J2=1,N3                                                     MK1 3030
      J3=J3+1                                                           MK1 3040
      B(J2)=BX(J3)                                                      MK1 3050
70    X(J2)=XX(J3)                                                      MK1 3060
80    CONTINUE                                                          MK1 3070
      GO TO 150                                                         MK1 3080
90    N4=NGROUP-2                                                       MK1 3090
      J3=2                                                              MK1 3100
      DO 110 J1=1,N4                                                    MK1 3110
      YB(J1)=ABETA(B,N4)                                                MK1 3120
      YX(J1)=ALAMDA(B,X,N4)                                             MK1 3130
      DO 100 J2=1,N4                                                    MK1 3140
      J3=J3+1                                                           MK1 3150
      B(J2)=BX(J3)                                                      MK1 3160
100   X(J2)=XX(J3)                                                      MK1 3170
110   CONTINUE                                                          MK1 3180
      DO 120 NO=3,NGROUP                                                MK1 3190
      N1=NO+2                                                           MK1 3200
      YB(NO)=BX(N1)                                                     MK1 3210
120   YX(NO)=BX(N1)                                                     MK1 3220
      GO TO 150                                                         MK1 3230
130   N5=NGROUP-1                                                       MK1 3240
      YB(1)=ABETA(B,2)                                                  MK1 3250
      YX(1)=ALAMDA(B,X,2)                                               MK1 3260
      DO 140 J1=2,NGROUP                                                MK1 3270
      J2=J1+1                                                           MK1 3280
      YB(J1)=BX(J2)                                                     MK1 3290
140   YX(J1)=XX(J2)                                                     MK1 3300
      GO TO 150                                                         MK1 3310
150   DO 160 J2=1,NGROUP                                                MK1 3320
      B(J2)=YB(J2)                                                      MK1 3330
160   X(J2)=YX(J2)                                                      MK1 3340
      RETURN                                                            MK1 3350
      END                                                               MK1 3360
C                                                                       MK1 3370
C                                                                       MK1 3380
C     ---------------FUNCTION ALAMDA IS AVERAGING THE VALUE OF LAMBDA   MK1 3390
C              SEE EQUATION 1.3.5                                       MK1 3400
C                                                                       MK1 3410
C                                                                       MK1 3420


      FUNCTION ALAMDA (B,X,N)                                           MK1 3430
      DIMENSION B(N), X(N)                                              MK1 3440
      B1=0.0                                                            MK1 3450
      BL=0.0                                                            MK1 3460
      DO 10 I=1,N                                                       MK1 3470
      B1=B1+B(I)                                                        MK1 3480
10    BL=BL+B(I)/X(I)                                                   MK1 3490
      ALAMDA=B1/BL                                                      MK1 3500
      RETURN                                                            MK1 3510
      END                                                               MK1 3520
C                                                                       MK1 3530
C                                                                       MK1 3540
C     ---------------FUNCTION ABETA IS AVERAGING BETA .                MK1 3550
C              SEE EQUATION 1.2.3                                       MK1 3560
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C                                                                       MK1 3570
C                                                                       MK1 3580


      FUNCTION ABETA (B,N)                                              MK1 3590
      DIMENSION B(N)                                                    MK1 3600
      ABETA=0.0                                                         MK1 3610
      DO 10 I=1,N                                                       MK1 3620
10    ABETA=ABETA+B(I)                                                  MK1 3630
      RETURN                                                            MK1 3640
      END                                                               MK1 3650
                                                                        MK1 3660
C                                                                       MK1 3670
C                                                                       MK1 3680
C-------------SUBROUTINE U-235 IS PROVIDING DATA FOR BETA AND LAMBDA   MK1 3690
C        FOR SIX GROUP DELAYED NEUTRON FROM THERMAL FISSION .           MK1 3700
C                                                                       MK1 3710
C


      SUBROUTINE U235 (B,X)                                             MK1 3720
      DIMENSION B(6), X(6)                                              MK1 3730
      B(1)=.00021                                                       MK1 3740
      B(2)=.00141                                                       MK1 3750
      B(3)=.00127                                                       MK1 3760
      B(4)=.00255                                                       MK1 3770
      B(5)=.00074                                                       MK1 3780
      B(6)=.00027                                                       MK1 3790
      X(1)=.0124                                                        MK1 3800
      X(2)=.0305                                                        MK1 3810
      X(3)=.111                                                         MK1 3820
      X(4)=.301                                                         MK1 3830
      X(5)=1.14                                                         MK1 3840
      X(6)=3.01                                                         MK1 3850
      RETURN                                                            MK1 3860
      END                                                               MK1 3870
                                                                        MK1 3880
C                                                                       MK1 3890
C                                                                       MK1 3900
C-------------SUBROUTINE PU-239 IS PROVIDING DATA FOR BETA AND LAMBDA   MK1 3910
C          DA SIX GROUP DELAYED NEUTRON FROM THERMAL FISSION .          MK1 3920
C                                                                       MK1 3930
C


      SUBROUTINE PU239 (B,X)                                            MK1 3940
      DIMENSION B(6), X(6)                                              MK1 3950
      B(1)=.00009450                                                    MK1 3960
      B(2)=.00030460                                                    MK1 3970
      B(3)=.00056970                                                    MK1 3980
      B(4)=.00088020                                                    MK1 3990
      B(5)=.00023220                                                    MK1 4000
      B(6)=.00011880                                                    MK1 4010
      X(1)=.0128                                                        MK1 4020
      X(2)=.0301                                                        MK1 4030
      X(3)=.124                                                         MK1 4040
      X(4)=.325                                                         MK1 4050
      X(5)=1.12                                                         MK1 4060
      X(6)=2.69                                                         MK1 4070
      RETURN                                                            MK1 4080
      END                                                               MK1 4090
                                                                        MK1 4100
C
```

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      C                                                                 MK1 4110
      C     --------------THE MAIN PROGRAM IS TRYING TO CALCULATE THE COEFFICIMK1 4120
      C           ENTS OF INHOUR FORMULA FOR ANY GROUP BETWEEN 1 TO 6 .     MK1 4130
      C                                                                 MK1 4140
      C                                                                 MK1 4150


      SUBROUTINE ANHOUR (XL,X,B,R,A,NN)                                 MK1 4160
      DIMENSION X(NN), B(NN), A(8)                                      MK1 4170
      BB=0.0                                                            MK1 4180
      DO 10 I=1,NN                                                      MK1 4190
   10 BB=BB+B(I)                                                        MK1 4200
      RO=R*BB                                                           MK1 4210
      JJ=NN+2                                                           MK1 4220
      DO 20 J=1,JJ                                                      MK1 4230
   20 A(J)=0.0                                                          MK1 4240
      C            SEE 1.13.13 FOR NN=1                                 MK1 4250
      GO TO (30,40,50,60,70,80), NN                                     MK1 4260
   30 CALL G1 (XL,X,B,RO,A)                                             MK1 4270
      GO TO 90                                                          MK1 4280
   40 CALL G2 (XL,X,B,RO,A)                                             MK1 4290
      GO TO 90                                                          MK1 4300
   50 CALL G3 (XL,X,B,RO,A)                                             MK1 4310
      GO TO 90                                                          MK1 4320
   60 CALL G4 (XL,X,B,RO,A)                                             MK1 4330
      GO TO 90                                                          MK1 4340
   70 CALL G5 (XL,X,B,RO,A)                                             MK1 4350
      GO TO 90                                                          MK1 4360
   80 CALL G6 (XL,X,B,RO,A)                                             MK1 4370
      GO TO 90                                                          MK1 4380
   90 RETURN                                                            MK1 4390
      END                                                               MK1 4400
      C                                                                 MK1 4410
      C                                                                 MK1 4420
      C     --------------SUBROUTINE G1,G2,G3,G4,G5,G6, IS  DIRECTLY CALCULATIMK1 4430
      C           COEFFICIENTS OF INHOUR FORMULA IN POLYNOMIAL FORM IN      MK1 4440
      C           THE ORDER OF THE SMALLEST DEGREE TO THE LARGEST .         MK1 4450
      C           THE FORM OUTPUT IS A1*W**N + A2*W**(N-1) + A3*W**(N-2)    MK1 4460
      C           +.....+ AN                                                MK1 4470
      C           SEE EQUATION 1.3.15 FOR THE GENERAL INHOUR FORMULA        MK1 4480
      C                                                                     MK1 4490
      C           SUPPORTING ROUTINE     A62,A52,A42,A32,A22  AND           MK1 4500
      C                                  XX,XY,XZ,XU,XV,AA .                MK1 4510
      C                                                                     MK1 4520
      C                                                                     MK1 4530


      SUBROUTINE G1 (XL,X,B,RO,W)                                       MK1 4540
      DIMENSION X(1), B(1), W(3)                                        MK1 4550
      W(3)=XL                                                           MK1 4560
      W(2)=B(1)+XL*X(1)-RO                                              MK1 4570
      W(1)=-RO*X(1)                                                     MK1 4580
      RETURN                                                            MK1 4590
      END                                                               MK1 4600


      SUBROUTINE G2 (XL,X,B,RO,W)                                       MK1 4610
      DIMENSION X(2), B(2), W(4)                                        MK1 4620
      W(4)=XL                                                           MK1 4630
      W(3)=XX(8,2)+XL*XX(X,2)-RO                                        MK1 4640
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      W(2)=XL*AA(X,2)+A62(B,X,2)-RO*XX(X,2)                          MK1 4650
      W(1)=-RO*AA(X,2)                                               MK1 4660
      RETURN                                                         MK1 4670
      END                                                            MK1 4680

      SUBROUTINE G3 (XL,X,B,RO,W)                                    MK1 4690
      DIMENSION X(3), B(3), W(5)                                     MK1 4700
      W(5)=XL                                                        MK1 4710
      W(4)=XX(B,3)+XL*XX(X,3)-RO                                     MK1 4720
      W(3)=XL*XY(X,3)+A62(B,X,3)-RO*XX(X,3)                          MK1 4730
      W(2)=XL*AA(X,3)+A52(B,X,3)-RO*XY(X,3)                          MK1 4740
      W(1)=-RO*AA(X,3)                                               MK1 4750
      RETURN                                                         MK1 4760
      END                                                            MK1 4770

      SUBROUTINE G4 (XL,X,B,RO,W)                                    MK1 4780
      DIMENSION X(4), B(4), W(6)                                     MK1 4790
      W(6)=XL                                                        MK1 4800
      W(5)=XX(B,4)+XL*XX(X,4)-RO                                     MK1 4810
      W(4)=XL*XY(X,4)+A62(B,X,4)-RO*XX(X,4)                          MK1 4820
      W(3)=XL*XZ(X,4)+A52(B,X,4)-RO*XY(X,4)                          MK1 4830
      W(2)=XL*AA(X,4)+A42(B,X,4)-RO*XZ(X,4)                          MK1 4840
      W(1)=-RO*AA(X,4)                                               MK1 4850
      RETURN                                                         MK1 4860
      END                                                            MK1 4870

      SUBROUTINE G5 (XL,X,B,RO,W)                                    MK1 4880
      DIMENSION X(5), B(5), W(7)                                     MK1 4890
      W(7)=XL                                                        MK1 4900
      W(6)=XX(B,5)+XL*XX(X,5)-RO                                     MK1 4910
      W(5)=XL*XY(X,5)+A62(B,X,5)-RO*XX(X,5)                          MK1 4920
      W(4)=XL*XZ(X,5)+A52(B,X,5)-RO*XY(X,5)                          MK1 4930
      W(3)=XL*XU(X,5)+A42(B,X,5)-RO*XZ(X,5)                          MK1 4940
      W(2)=XL*AA(X,5)+A32(B,X,5)-RO*XU(X,5)                          MK1 4950
      W(1)=-RO*AA(X,5)                                               MK1 4960
      RETURN                                                         MK1 4970
      END                                                            MK1 4980

      SUBROUTINE G6 (XL,X,B,RO,W)                                    MK1 4990
      DIMENSION X(6), B(6), W(8)                                     MK1 5000
      W(8)=XL                                                        MK1 5010
      W(7)=XX(B,6)+XL*XX(X,6)-RO                                     MK1 5020
      W(6)=XL*XY(X,6)+A62(B,X,6)-RO*XX(X,6)                          MK1 5030
      W(5)=XL*XZ(X,6)+A52(B,X,6)-RO*XY(X,6)                          MK1 5040
      W(4)=XL*XU(X,6)+A42(B,X,6)-RO*XZ(X,6)                          MK1 5050
      W(3)=XL*XV(X,6)+A32(B,X,6)-RO*XU(X,6)                          MK1 5060
      W(2)=XL*AA(X,6)+A22(B,X,6)-RO*XV(X,6)                          MK1 5070
      W(1)=-RO*AA(X,6)                                               MK1 5080
      RETURN                                                         MK1 5090
      END                                                            MK1 5100
C                                                                    MK1 5110
C                                                                    MK1 5120
C     ----------FUNCTION A62 FUNCTION AS :                            MK1 5130
C           A62 = B1*(X2+X3+X4+...) + B2*(B1+B3+B4+....) +           MK1 5140
C           B3*(B1+B2+B3+B4+...) + .....   IN THAT COMBINATIONS .    MK1 5150
C                                                                    MK1 5160
```

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      C                                                                 MK1 5170


308         FUNCTION A62 (B,X,M)                                        MK1 5180
309         DIMENSION X(M), AX1(6), B(M), W(6)                          MK1 5190
310         A62=0.0                                                     MK1 5200
311         DO 20 I=1,M                                                 MK1 5210
312         DO 10 J=1,M                                                 MK1 5220
313   10    AX1(J)=X(J)                                                 MK1 5230
314         AX1(I)=0.0                                                  MK1 5240
315         W(I)=B(I)*XX(AX1,M)                                         MK1 5250
316   20    A62=A62+W(I)                                                MK1 5260
317         RETURN                                                      MK1 5270
318         END                                                         MK1 5280
      C                                                                 MK1 5290
      C                                                                 MK1 5300
      C     --------------A52 IS ALMOST THE SAME THING AS A62 IN IT'S FUNCTIONMK1 5310
      C               EXCEPT THAT X1 = Y1*Y2 WHICH THE POSTSCRIPT NEVER BE MK1 5320
      C               EQUAL AND THE FIRST CHARACTER POSTSCRIPT IS ALWAYS   MK1 5330
      C               SMALLER THAN THE NEXT .                             MK1 5340
      C                                                                   MK1 5350
      C                                                                   MK1 5360


319         FUNCTION A52 (B,X,M)                                        MK1 5370
320         DIMENSION X(M), AX2(6), B(M), W(6)                          MK1 5380
321         A52=0.0                                                     MK1 5390
322         DO 20 I=1,M                                                 MK1 5400
323         DO 10 J=1,M                                                 MK1 5410
324   10    AX2(J)=X(J)                                                 MK1 5420
325         AX2(I)=0.0                                                  MK1 5430
326         W(I)=B(I)*XY(AX2,M)                                         MK1 5440
327   20    A52=A52+W(I)                                                MK1 5450
328         RETURN                                                      MK1 5460
329         END                                                         MK1 5470
      C                                                                 MK1 5480
      C                                                                 MK1 5490
      C     --------------DED AS A62 EXCEPT  ITS ADDITION OF THREE CHARACTER .MK1 5500
      C                                                                 MK1 5510
      C                                                                 MK1 5520


330         FUNCTION A42 (B,X,M)                                        MK1 5530
331         DIMENSION X(M), AX3(6), B(M), W(6)                          MK1 5540
332         A42=0.0                                                     MK1 5550
333         DO 20 I=1,M                                                 MK1 5560
334         DO 10 J=1,M                                                 MK1 5570
335   10    AX3(J)=X(J)                                                 MK1 5580
336         AX3(I)=0.0                                                  MK1 5590
337         W(I)=B(I)*XZ(AX3,M)                                         MK1 5600
338   20    A42=A42+W(I)                                                MK1 5610
339         RETURN                                                      MK1 5620
340         END                                                         MK1 5630
      C                                                                 MK1 5640
      C                                                                 MK1 5650
      C     --------------DED AS A62 EXCEPT CHARACTER ADDITION IS FOUR .  MK1 5660
      C                                                                 MK1 5670
      C                                                                 MK1 5680
```

```
      FUNCTION A32(B,X,M)                                                MK1 5690
      DIMENSION X(M), AX4(6), B(M), W(6)                                 MK1 5700
      A32=0.0                                                            MK1 5710
      DO 20 I=1,M                                                        MK1 5720
      DO 10 J=1,M                                                        MK1 5730
10    AX4(J)=X(J)                                                        MK1 5740
      AX4(I)=0.0                                                         MK1 5750
      W(I)=B(I)*XU(AX4,M)                                                MK1 5760
20    A32=A32+W(I)                                                       MK1 5770
      RETURN                                                             MK1 5780
      END                                                                MK1 5790
C                                                                        MK1 5800
C                                                                        MK1 5810
C     ----------UED AS A62 EXCEPT ADDITION CHARACTER IS FIVE .           MK1 5820
C                                                                        MK1 5830
C                                                                        MK1 5840


      FUNCTION A22 (B,X,M)                                               MK1 5850
      DIMENSION X(M), AX5(6), B(M), W(6)                                 MK1 5860
      A22=0.0                                                            MK1 5870
      DO 20 I=1,M                                                        MK1 5880
      DO 10 J=1,M                                                        MK1 5890
10    AX5(J)=X(J)                                                        MK1 5900
      AX5(I)=0.0                                                         MK1 5910
      W(I)=B(I)*XV(AX5,M)                                                MK1 5920
20    A22=A22+W(I)                                                       MK1 5930
      RETURN                                                             MK1 5940
      END                                                                MK1 5950
C                                                                        MK1 5960
C                                                                        MK1 5970
C     ----------XX IS COMPUTING THE TOTAL OF X                           MK1 5980
C           XX = (X1+X2+X3+X4+.....+XM)                                  MK1 5990
C                                                                        MK1 6000
C                                                                        MK1 6010


      FUNCTION XX (X,M)                                                  MK1 6020
      DIMENSION X(M)                                                     MK1 6030
      XX=0.0                                                             MK1 6040
      DO 10 I=1,M                                                        MK1 6050
10    XX=XX+X(I)                                                         MK1 6060
      RETURN                                                             MK1 6070
      END                                                                MK1 6080
C                                                                        MK1 6090
C                                                                        MK1 6100
C     ----------XY IS COMPUTING THE ADDITION OF TWO PRODUCES .          MK1 6110
C           XY = (X1*X2+X1*X3+X1*X4+.....+X2*X3+X2*X4+.....)             MK1 6120
C           THE FIRST POSTCRIPT ALWAYS SMALLER THAN THE NEXT             MK1 6130
C                                                                        MK1 6140
C                                                                        MK1 6150


      FUNCTION XY (X,M)                                                  MK1 6160
      DIMENSION X(M)                                                     MK1 6170
      XY=0.0                                                             MK1 6180
      DO 20 I=1,M                                                        MK1 6190
      DO 20 J=1,M                                                        MK1 6200
      A=X(I)                                                             MK1 6210
      B=X(J)                                                             MK1 6220
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      IF (I-J) 10,20,20                                                 MK1 6230
10    Y=A*B                                                             MK1 6240
      XY=XY+Y                                                           MK1 6250
20    CONTINUE                                                          MK1 6260
      RETURN                                                            MK1 6270
      END                                                               MK1 6280
C                                                                       MK1 6290
C                                                                       MK1 6300
C     --------------QED AS XY EXCEPT IT IS A PRODUCT OF THREE .         MK1 6310
C                                                                       MK1 6320
C                                                                       MK1 6330


      FUNCTION XZ (X,M)                                                 MK1 6340
      DIMENSION X(M)                                                    MK1 6350
      XZ=0.0                                                            MK1 6360
      DO 30 I=1,M                                                       MK1 6370
      DO 30 J=1,M                                                       MK1 6380
      DO 30 K=1,M                                                       MK1 6390
      A=X(I)                                                            MK1 6400
      B=X(J)                                                            MK1 6410
      C=X(K)                                                            MK1 6420
      IF (I-J) 10,30,30                                                 MK1 6430
10    IF (J-K) 20,30,30                                                 MK1 6440
20    Y=A*B*C                                                           MK1 6450
      XZ=XZ+Y                                                           MK1 6460
30    CONTINUE                                                          MK1 6470
      RETURN                                                            MK1 6480
      END                                                               MK1 6490
C                                                                       MK1 6500
C                                                                       MK1 6510
C     --------------QED AS XY EXCEPT FOR PRODUCT OF FOUR .             MK1 6520
C                                                                       MK1 6530
C                                                                       MK1 6540


      FUNCTION XU (X,M)                                                 MK1 6550
      DIMENSION X(M)                                                    MK1 6560
      XU=0.0                                                            MK1 6570
      DO 40 I=1,M                                                       MK1 6580
      DO 40 J=1,M                                                       MK1 6590
      DO 40 K=1,M                                                       MK1 6600
      DO 40 L=1,M                                                       MK1 6610
      A=X(I)                                                            MK1 6620
      B=X(J)                                                            MK1 6630
      C=X(K)                                                            MK1 6640
      D=X(L)                                                            MK1 6650
      IF (I-J) 10,40,40                                                 MK1 6660
10    IF (J-K) 20,40,40                                                 MK1 6670
20    IF (K-L) 30,40,40                                                 MK1 6680
30    Y=A*B*C*D                                                         MK1 6690
      XU=XU+Y                                                           MK1 6700
40    CONTINUE                                                          MK1 6710
      RETURN                                                            MK1 6720
      END                                                               MK1 6730
C                                                                       MK1 6740
C                                                                       MK1 6750
C     --------------QED EXCEPT ITS AN ADDITION OF MULTIPLICATION OF FIVEMK1 6760
C                                                                       MK1 6770
C                                                                       MK1 6780
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```
      FUNCTION XV (X,M)                                                 MK1 6790
      DIMENSION X(M)                                                    MK1 6800
      XV=0.0                                                            MK1 6810
      DO 50 I=1,M                                                       MK1 6820
      DO 50 J=1,M                                                       MK1 6830
      DO 50 K=1,M                                                       MK1 6840
      DO 50 L=1,M                                                       MK1 6850
      DO 50 N=1,M                                                       MK1 6860
      A=X(I)                                                            MK1 6870
      B=X(J)                                                            MK1 6880
      C=X(K)                                                            MK1 6890
      D=X(L)                                                            MK1 6900
      E=X(N)                                                            MK1 6910
      IF (I-J) 10,50,50                                                 MK1 6920
10    IF (J-K) 20,50,50                                                 MK1 6930
20    IF (K-L) 30,50,50                                                 MK1 6940
30    IF (L-N) 40,50,50                                                 MK1 6950
40    Y=A*B*C*D*E                                                       MK1 6960
      XV=XV+Y                                                           MK1 6970
50    CONTINUE                                                          MK1 6980
      WRITE (6,60) XV                                                   MK1 6990
60    FORMAT (2X,7H XV  = ,F19.10)                                      MK1 7000
      RETURN                                                            MK1 7010
      END                                                               MK1 7020
C                                                                       MK1 7030
C                                                                       MK1 7040
C     --------------AA IS COMPUTING MULTIPLICATION OF N CHARACTER .     MK1 7050
C             AA = X1*X2*X3*X4*.............*XN                        MK1 7060
C                                                                       MK1 7070
C                                                                       MK1 7080


      FUNCTION AA (X,M)                                                 MK1 7090
      DIMENSION X(M)                                                    MK1 7100
      AA=X(1)                                                           MK1 7110
      DO 10 I=2,M                                                       MK1 7120
10    AA=AA*X(I)                                                        MK1 7130
      RETURN                                                            MK1 7140
      END                                                               MK1 7150
C                                                                       MK1 7160
C                                                                       MK1 7170
C     --------------FORM A X N MATRIX BY HANSEN METHOD , G = MATRIX .  MK1 7180
C             THE FIRST COLUMN , THE FIRST ROW AND THE DIAGONAL .NE 0,  MK1 7190
C             THE REST OF THEM = 0 .                                    MK1 7200
C             SEE EQUATION 1.4.15                                       MK1 7210
C                                                                       MK1 7220
C             SUPPORTING ROUTINE  NONE .                                MK1 7230
C                                                                       MK1 7240
C                                                                       MK1 7250


      SUBROUTINE GMTRX (P,R,X,XL,WO,N,P,G)                              MK1 7260
      DIMENSION R(6), X(6), G(N,N)                                      MK1 7270
      GD(AX,H)=EXP(-AX*H)                                               MK1 7280
      GH(RBL,WO,H,AX)=((EXP(WO*H)-EXP(RBL*H))/(WO-RBL))*AX              MK1 7290
      GV(XL,WO,H,AX,BX)=((EXP(WO*H)-GD(AX,H))*BX)/((WO+AX)*XL)          MK1 7300
      RB(R,BX)=(R-BX)/XL                                                MK1 7310
      BB=0.0                                                            MK1 7320
      NN=N-1                                                            MK1 7330
      DO 10 I=1,NN                                                      MK1 7340
```

```
458   10    BB=BB+B(I)                                                    MK1 7350
459         DO 150 J=1,N                                                  MK1 7360
460         DO 140 JJ=1,N                                                 MK1 7370
461         J1=JJ                                                         MK1 7380
462         RBL=RB(R,BB)                                                  MK1 7390
463         IF (J-JJ) 20,60,20                                            MK1 7400
464   20    IF (J-1) 30,50,30                                             MK1 7410
465   30    J1=JJ+1                                                       MK1 7420
466         IF (JJ-N) 50,40,50                                            MK1 7430
467   40    IF (J-N) 120,80,120                                           MK1 7440
468   50    AX=X(J1-1)                                                    MK1 7450
469         GO TO 90                                                      MK1 7460
470   60    IF (J-1) 80,70,80                                             MK1 7470
471   70    G(I,1)=EXP(RB(R,BB)*H)                                        MK1 7480
472         GO TO 140                                                     MK1 7490
473   80    AX=X(J-1)                                                     MK1 7500
474         G(J,JJ)=GD(AX,H)                                              MK1 7510
475         GO TO 140                                                     MK1 7520
476   90    IF (J-1) 110,100,110                                          MK1 7530
477   100   G(1,JJ)=GH(RBL,WO,H,AX)                                       MK1 7540
478         GO TO 140                                                     MK1 7550
479   110   IF (JJ-1) 120,130,120                                         MK1 7560
480   120   G(J,JJ)=0.0                                                   MK1 7570
481         GO TO 140                                                     MK1 7580
482   130   AX=X(J-1)                                                     MK1 7590
483         BX=B(J-1)                                                     MK1 7600
484         G(J,1)=GV(XL,WO,H,AX,BX)                                      MK1 7610
485   140   CONTINUE                                                      MK1 7620
486         I=J                                                           MK1 7630
      C     WRITE(6,144)(G(I,II),II=1,N)                                  MK1 7640
      C 144 FORMAT('***',2(E10.3,2X),'***')                               MK1 7650
487   150   CONTINUE                                                      MK1 7660
488         RETURN                                                        MK1 7670
489         END                                                           MK1 7680
      C                                                                   MK1 7690
      C                                                                   MK1 7700
      C     ----------MULTIPLY  THE G - MATRIX WITH THE INITIAL VECTOR COLMK1 7710
      C               TO GET THE NEXT ONE , ITERATION .                   MK1 7720
      C               SEE EQUATION 1.4.3                                  MK1 7730
      C                                                                   MK1 7740
      C                                                                   MK1 7750


490         SUBROUTINE GXN (G,XN,N,GX)                                    MK1 7760
491         DIMENSION G(N,N), XN(N), GX(N)                                MK1 7770
492         DO 20 I=1,N                                                   MK1 7780
493         GG=0.0                                                        MK1 7790
494         DO 10 J=1,N                                                   MK1 7800
495   10    GG=GG+G(I,J)*XN(J)                                            MK1 7810
496   20    GX(I)=GG                                                      MK1 7820
497         RETURN                                                        MK1 7830
498         END                                                           MK1 7840
      C                                                                   MK1 7850
      C                                                                   MK1 7860
      C                                                                   MK1 7870
      C        SUBROUTINE POLRT                                           MK1 7880
      C                                                                   MK1 7890
      C        PURPOSE                                                    MK1 7900
      C           COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL MK1 7910
      C                                                                   MK1 7920
```

```
C       USAGE                                                          MK1 7930
C          CALL POLRT(XCOF,COF,M,ROOTR,ROOTI,IER,M1)                   MK1 7940
C                                                                      MK1 7950
C       DESCRIPTION OF PARAMETERS                                      MK1 7960
C          XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL          MK1 7970
C               ORDERED FROM SMALLEST TO LARGEST POWER                 MK1 7980
C          COF  -WORKING VECTOR OF LENGTH M+1                          MK1 7990
C          M    -ORDER OF POLYNOMIAL                                   MK1 8000
C          ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS    MK1 8010
C               OF THE POLYNOMIAL                                      MK1 8020
C          ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE           MK1 8030
C               CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL        MK1 8040
C          IER  -ERROR CODE WHERE                                      MK1 8050
C               IER=0  NO ERROR                                        MK1 8060
C               IER=1  M LESS THAN ONE                                 MK1 8070
C               IER=2  M GREATER THAN 36                               MK1 8080
C               IER=3  UNABLE TO DETERMINE ROOT WITH 500 ITERATIONS    MK1 8090
C                      ON 5 STARTING VALUES                            MK1 8100
C               IER=4  HIGH ORDER COEFFICIENT IS ZERO                  MK1 8110
C          M1   -NUMBER OF COEFFICIENT , M+1                           MK1 8120
C                                                                      MK1 8130
C       REMARKS                                                        MK1 8140
C          LIMITED TO 36TH,ORDER POLYNOMIAL OR LESS.                   MK1 8150
C          FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER            MK1 8160
C          POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE         MK1 8170
C          RESULTS .                                                   MK1 8180
C                                                                      MK1 8190
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  MK1 8200
C          NONE                                                        MK1 8210
C                                                                      MK1 8220
C       METHOD                                                         MK1 8230
C          NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS   MK1 8240
C          ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL    MK1 8250
C          RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED     MK1 8260
C          ERRORS IN THE REDUCED POLYNOMIAL.                           MK1 8270
C                                                                      MK1 8280
C                                                                      MK1 8290


499        SUBROUTINE POLRT (XCOF,COF,M,ROOTR,ROOTI,IER,M1)            MK1 8300
500        DIMENSION XCOF(M1), COF(M1), ROOTR(M), ROOTI(M)             MK1 8310
C          DOUBLE PRECISION XO,YO,X,Y,XPR,YPP,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,MK1 8320
C          1 DX,DY,TEMP,ALPHA,DABS                                     MK1 8330
C                                                                      MK1 8340
C                                                                      MK1 8350
C          IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THEMK1 8360
C          C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION   MK1 8370
C          STATEMENT WHICH FOLLOWS.                                    MK1 8380
C                                                                      MK1 8390
C       DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI                          MK1 8400
C                                                                      MK1 8410
C          THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS MK1 8420
C          APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS   MK1 8430
C          ROUTINE.                                                    MK1 8440
C          THE DOUBLE PRECISION VERSION MAY BE MODIFIED BY CHANGING THEMK1 8450
C          CONSTANT IN STATEMENT 78 TO 1.0D-12 AND IN STATEMENT 122 TO MK1 8460
C          1.0D-10.  THIS WILL PROVIDE HIGHER PRECISION RESULTS AT THE MK1 8470
C          COST OF EXECUTION TIME                                      MK1 8480
C                                                                      MK1 8490
C                                                                      MK1 8500
```

```
501           IFIT=0                                                  MK1 8510
502           N=M                                                     MK1 8520
503           IER=0                                                   MK1 8530
504           IF (XCOF(N+1)) 10,40,10                                 MK1 8540
505     10    IF (N) 20,20,60                                         MK1 8550
              C                                                       MK1 8560
              C       SET ERROR CODE TO 1                             MK1 8570
              C                                                       MK1 8580
506     20    IER=1                                                   MK1 8590
507     30    RETURN                                                  MK1 8600
              C                                                       MK1 8610
              C       SET ERROR CODE TO 4                             MK1 8620
              C                                                       MK1 8630
508     40    IER=4                                                   MK1 8640
509           GO TO 30                                                MK1 8650
              C                                                       MK1 8660
              C       SET ERROR CODE TO 2                             MK1 8670
              C                                                       MK1 8680
510     50    IER=2                                                   MK1 8690
511           GO TO 30                                                MK1 8700
512     60    IF (N-36) 70,70,50                                      MK1 8710
513     70    NX=N                                                    MK1 8720
514           NXX=N+1                                                 MK1 8730
515           N2=1                                                    MK1 8740
516           KJ1=N+1                                                 MK1 8750
517           DO 80 L=1,KJ1                                           MK1 8760
518           MT=KJ1-L+1                                              MK1 8770
519     80    COF(MT)=XCOF(L)                                         MK1 8780
              C                                                       MK1 8790
              C       SET INITIAL VALUES                              MK1 8800
              C                                                       MK1 8810
520     90    X0=.00500101                                            MK1 8820
521           Y0=0.01000101                                           MK1 8830
              C                                                       MK1 8840
              C       ZERO INITIAL VALUE COUNTER                      MK1 8850
              C                                                       MK1 8860
522           IN=0                                                    MK1 8870
523     100   X=X0                                                    MK1 8880
              C                                                       MK1 8890
              C       INCREMENT INITIAL VALUES AND COUNTER            MK1 8900
              C                                                       MK1 8910
524           X0=-10.0*Y0                                             MK1 8920
525           Y0=-10.0*X                                              MK1 8930
              C                                                       MK1 8940
              C       SET X AND Y TO CURRENT VALUE                    MK1 8950
              C                                                       MK1 8960
526           X=X0                                                    MK1 8970
527           Y=Y0                                                    MK1 8980
528           IN=IN+1                                                 MK1 8990
529           GO TO 120                                               MK1 9000
530     110   IFIT=1                                                  MK1 9010
531           XPR=X                                                   MK1 9020
532           YPR=Y                                                   MK1 9030
              C                                                       MK1 9040
              C       EVALUATE POLYNOMIAL AND DERIVATIVES             MK1 9050
              C                                                       MK1 9060
533     120   ICT=0                                                   MK1 9070
534     130   UX=0.0                                                  MK1 9080
535           UY=0.0                                                  MK1 9090
536           V=0.0                                                   MK1 9100
```

```
537           YT=0.0                                                  MK1 9110
538           XT=1.0                                                  MK1 9120
539           U=COF(N+1)                                              MK1 9130
540           IF (U) 140,270,140                                      MK1 9140
541     140   DO 150 I=1,N                                            MK1 9150
542           L=N-I+1                                                 MK1 9160
543           TEMP=COF(L)                                             MK1 9170
544           XT2=X*XT-Y*YT                                           MK1 9180
545           YT2=X*YT+Y*XT                                           MK1 9190
546           U=U+TEMP*XT2                                            MK1 9200
547           V=V+TEMP*YT2                                            MK1 9210
548           FI=I                                                    MK1 9220
549           UX=UX+FI*XT*TEMP                                        MK1 9230
550           UY=UY-FI*YT*TEMP                                        MK1 9240
551           XT=XT2                                                  MK1 9250
552     150   YT=YT2                                                  MK1 9260
553           SUMSQ=UX*UX+UY*UY                                       MK1 9270
554           IF (SUMSQ) 160,230,160                                  MK1 9280
555     160   DX=(V*UY-U*UX)/SUMSQ                                    MK1 9290
556           X=X+DX                                                  MK1 9300
557           DY=-(U*UY+V*UX)/SUMSQ                                   MK1 9310
558           Y=Y+DY                                                  MK1 9320
559           IF (ABS(DY)+ABS(DX)-1.0E-5) 210,170,170                 MK1 9330
      C                                                               MK1 9340
      C        STEP ITERATION COUNTER                                 MK1 9350
      C                                                               MK1 9360
560     170   ICT=ICT+1                                               MK1 9370
561           IF (ICT-500) 130,180,180                                MK1 9380
562     180   IF (IFIT) 210,190,210                                   MK1 9390
563     190   IF (IN-5) 100,200,200                                   MK1 9400
      C                                                               MK1 9410
      C        SET ERROR CODE TO 3                                    MK1 9420
      C                                                               MK1 9430
564     200   IER=3                                                   MK1 9440
565           GO TO 30                                                MK1 9450
566     210   DO 220 L=1,NXX                                          MK1 9460
567           MT=NXX-L+1                                              MK1 9470
568           TEMP=XCOF(MT)                                           MK1 9480
569           XCOF(MT)=COF(L)                                         MK1 9490
570     220   COF(L)=TEMP                                             MK1 9500
571           ITEMP=N                                                 MK1 9510
572           N=NX                                                    MK1 9520
573           NX=ITEMP                                                MK1 9530
574           IF (IFIT) 250,110,250                                   MK1 9540
575     230   IF (IFIT) 240,100,240                                   MK1 9550
576     240   X=XPR                                                   MK1 9560
577           Y=YPR                                                   MK1 9570
578     250   IFIT=0                                                  MK1 9580
579           IF (ABS(Y)-1.0E-4*ABS(X)) 280,260,260                   MK1 9590
580     260   ALPHA=X+X                                               MK1 9600
581           SUMSQ=X*X+Y*Y                                           MK1 9610
582           N=N-2                                                   MK1 9620
583           GO TO 290                                               MK1 9630
584     270   X=0.0                                                   MK1 9640
585           NX=NX-1                                                 MK1 9650
586           NXX=NXX-1                                               MK1 9660
587     280   Y=0.0                                                   MK1 9670
588           SUMSQ=0.0                                               MK1 9680
589           ALPHA=X                                                 MK1 9690
590           N=N-1                                                   MK1 9700
```

```
591   290   COF(2)=COF(2)+ALPHA*COF(1)                                        MK1 9710
592         IF (N.EQ.0) GO TO 310                                             MK1 9720
593         DO 300 L=2,N                                                      MK1 9730
594   300   COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)                     MK1 9740
595   310   ROOTI(N2)=Y                                                       MK1 9750
596         ROOTR(N2)=X                                                       MK1 9760
597         N2=N2+1                                                           MK1 9770
598         IF (SUMSQ) 320,330,320                                            MK1 9780
599   320   Y=-Y                                                              MK1 9790
600         SUMSQ=0.0                                                         MK1 9800
601         GO TO 310                                                         MK1 9810
602   330   IF (N) 30,30,90                                                   MK1 9820
603         END                                                               MK1 9830
      C                                                                       MK1 9840
      C                                                                       MK1 9850
      C                                                                       MK1 9860
      C     --------SUBROUTINE TPLOT  IS SINGLE PRECISION PLOTTING 5 DIFFERENTMK1 9870
      C        VARIABLES , ASSUMING CONSTANT TIME INCREMENT BETWEEN TWO      MK1 9880
      C        EVENTS .                                                      MK1 9890
      C                                                                      MK1 9900
      C        SUPPORTING ROUTINE NONE                                       MK1 9910
      C                                                                      MK1 9920
      C                                                                      MK1 9930
      C                                                                      MK1 9940


604         SUBROUTINE TPLOT (M1,M2,M8,M9,M0,JX)                            MK1 9950
605         IMPLICIT REAL*4(A-H,M-Z)                                        MK1 9960
606         DIMENSION M8(JX), M9(JX), M0(JX), M1(JX), M2(JX)                MK1 9970
607         DIMENSION LINE(61), INUM(9)                                     MK1 9980
608         INTEGER PL,MI,ST,BL,SL,S9,S0,S6,S1,S2                           MK1 9990
609         READ (5,80) PL,MI,ST,BL,SL,S9,S0,S1,S2,S6                       MK110000
610         MXY=0.0                                                         MK110010
611         MIN0=0.0                                                        MK110020
612         MIN2=0.0                                                        MK110030
613         MIN8=0.0                                                        MK110040
614         MIN9=0.0                                                        MK110050
615         PHI0=0.0                                                        MK110060
616         PHI2=0.0                                                        MK110070
617         PHI8=0.0                                                        MK110080
618         PHI9=0.0                                                        MK110090
619         DO 10 I=1,JX                                                    MK110100
620         IF (MIN0.GT.M0(I)) MIN0=M0(I)                                   MK110110
621         IF (MIN2.GT.M2(I)) MIN2=M2(I)                                   MK110120
622         IF (MIN8.GT.M8(I)) MIN8=M8(I)                                   MK110130
623         IF (MIN9.GT.M9(I)) MIN9=M9(I)                                   MK110140
624         IF (ABS(M0(I)).GT.PHI0) PHI0=ABS(M0(I))                         MK110150
625         IF (ABS(M2(I)).GT.PHI2) PHI2=ABS(M2(I))                         MK110160
626         IF (ABS(M8(I)).GT.PHI8) PHI8=ABS(M8(I))                         MK110170
627         IF (ABS(M9(I)).GT.PHI9) PHI9=ABS(M9(I))                         MK110180
628   10    CONTINUE                                                        MK110190
629         JJ=JX                                                           MK110200
630         JJ0=JJ*6+1                                                      MK110210
631         JJ1=JJ+1                                                        MK110220
632         WRITE (6,90)                                                    MK110230
633         WRITE (6,100)                                                   MK110240
634         PHI0=PHI0+ABS(MIN0)                                             MK110250
635         PHI2=PHI2+ABS(MIN2)                                             MK110260
636         PHI8=PHI8+ABS(MIN8)                                             MK110270
637         PHI9=PHI9+ABS(MIN9)                                             MK110280
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```
      DO 20 I=1,JJ                                                      MK110290
      IF (MIN0.LT.0.0) M0(I)=M0(I)+ABS(MIN0)                            MK110300
      IF (MIN2.LT.0.0) M2(I)=M2(I)+ABS(MIN2)                            MK110310
      IF (MIN8.LT.0.0) M8(I)=M8(I)+ABS(MIN8)                            MK110320
      IF (MIN9.LT.0.0) M9(I)=M9(I)+ABS(MIN9)                            MK110330
      M0(I)=M0(I)/PHI0                                                  MK110340
      M2(I)=M2(I)/PHI2                                                  MK110350
      M8(I)=M8(I)/PHI8                                                  MK110360
20    M9(I)=M9(I)/PHI9                                                  MK110370
      DO 30 I=1,9                                                       MK110380
30    INUM(I)=I                                                         MK110390
      WRITE (6,110) (INUM(I),I=1,9)                                     MK110400
      DO 70 I=1,JJ1                                                     MK110410
      IF (I.EQ.1) GO TO 50                                              MK110420
      MXY=M1(I-1)                                                       MK110430
      IP8=M8(I-1)*60+1.0                                                MK110440
      IP9=M9(I-1)*60+1.0                                                MK110450
      IP0=M0(I-1)*60+1.0                                                MK110460
      IP2=M2(I-1)*60+1.0                                                MK110470
      DO 40 I1=1,56,5                                                   MK110480
      LINE(I1)=BL                                                       MK110490
      DO 40 I2=1,5                                                      MK110500
      I3=I1+I2                                                          MK110510
      IF (I1.EQ.IP0) LINE(I1)=S0                                        MK110520
      IF (I3.EQ.IP0) LINE(I3)=S0                                        MK110530
      IF (I1.EQ.IP2) LINE(I1)=S2                                        MK110540
      IF (I3.EQ.IP2) LINE(I3)=S2                                        MK110550
      IF (I1.EQ.IP8) LINE(I1)=S1                                        MK110560
      IF (I3.EQ.IP8) LINE(I3)=S1                                        MK110570
      IF (I1.EQ.IP9) LINE(I1)=S9                                        MK110580
      IF (I3.EQ.IP9) LINE(I3)=S9                                        MK110590
40    CONTINUE                                                          MK110600
      LINE(61)=PL                                                       MK110610
      I1L=I-1                                                           MK110620
      IF(1 .EQ.IP0) LINE(1)=S0                                          MK110630
      IF(1 .EQ.IP2) LINE(1)=S2                                          MK110640
      IF(1 .EQ.IP8) LINE(1)=S1                                          MK110650
      IF(I .EQ.IP9) LINE(1)=S9                                          MK110660
      IF(IP0.EQ.61)LINE(61)=S0                                          MK110670
      IF(IP2.EQ.61)LINE(61)=S2                                          MK110680
      IF(IP8.EQ.61)LINE(61)=S1                                          MK110690
      IF(IP9.EQ.61)LINE(61)=S9                                          MK110700
      IF(IP9.NE.1.OR.IP8.NE.1.OR.IP0.NE.1.OR.IP2.NE.1)LINE(1)=PL        MK110710
      WRITE(6,120)MXY,(LINE(KK),KK=1,61)                                MK110720
      IF(I.EQ.JJ1) GO TO 70                                             MK110730
50    CONTINUE                                                          MK110740
      DO 60 I1=1,56,5                                                   MK110750
      DO 60 I2=1,5                                                      MK110760
      I3=I1+I2                                                          MK110770
      LINE(I3)=BL                                                       MK110780
60    CONTINUE                                                          MK110790
70    CONTINUE                                                          MK110800
      WRITE(6,130)(INUM(I),I=1,9)                                       MK110810
      WRITE(6,150)PL,MI,SF,BL,SL,S9,S0,S1,S2,S6                         MK110820
      WRITE(6,160)                                                      MK110830
      WRITE (6,140)                                                     MK110840
80    FORMAT(1IA1)                                                      MK110850
90    FORMAT('1')                                                       MK110860
100   FORMAT(35X,'RELATIVE DENSITY')                                    MK110870
110   FORMAT (17X,9(2X,'0.',I1,' ' )/,14X,10('+-----'),'+')            MK110880
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698    120   FORMAT(1X,'TIME',1X,F7.3,1X,61A1)                                  MK110890
699    130   FORMAT(14X,10('+-----'),'+',/,16X,9(3X,'0.',I1))                   MK110900
700    140   FORMAT ('1')                                                       MK110910
701    150   FORMAT(15X,'INPUT CHARACTER  ',11A1,/)                             MK110920
702    160   FORMAT('                                ',/,15X,' P - POWER',/,15X,' L -MK110930
            < LOG. OF POWER',/,15X,' Q - PRECURSOR DENSITY',/,15X,' R - REACTIVMK110940
            <ITY',///,15X,'NEGATIVE VALUE PLOTTED WITH THE AXIS IN THE CENTER',MK110950
                                                                               MK110960
            </////)                                                           MK110970
703          STOP                                                              MK110980
704          END
```

//DATA

********** KINETICS MODULE 1 **********

****
INPUT PARAMETER
****

TYPE OF FUEL U-239

NUMBER OF DELAYED NEUTRON = 3

NEUTRON GENERATION TIME = 0.00010 (SEC)

TYPE OF REACTIVITY = 2

RO = $ 0.250 B2 = 5.00000

TOTAL TIME USED = 1.000 (SEC) , TIME INTERVAL = 0.010 (SEC)

OUTPUT OPTION = 2

INITIAL RELATIVE FLUX = 1.000

********* END OF INPUT DATA *********

SIX GROUP LAMBDA ARE 0.0128 0.0301 0.1240 0.3250 1.1200 2.6900

SIX GROUP BETA ARE 0.00009 0.00080 0.00057 0.00088 0.00023 0.00012

THE AVERAGE BETA AND LAMDA

B(1)= 0.00090 LAMBDA(1) = 0.02636

B(2)= 0.00145 LAMBDA(2) = 0.19854

B(3)= 0.00035 LAMBDA(3) = 1.39571

COEFFICIENTS OF INHOUR FORMULA

A(1) = -0.0000E 00

A(2) = 0.3051E-03

A(3) = 0.3606E-02

A(4) = 0.2862E-02

A(5) = 0.1000E-03

THE LARGEST EIGEN-VALUE = 0.0000E 00

NO ( )	TIME (SEC)	R ($)	POWER (-)	3 GROUP DELAYED NEUTRON (RELATIVE)		
1	0.000	0.00	0.1000E 01	0.3411E 03	0.7303E 02	0.2515E 01

THE G-MATRIX

0.7634E 00 0.2310E-03 0.1740E-02 0.1223E-01

0.8991E-01 0.9997E 00 0.0000E 00 0.0000E 00

0.1448E 00 0.0000E 00 0.9980E 00 0.0000E 00

0.3486E-01 0.0000E 00 0.0000E 00 0.9861E 00

2	0.000	0.00	0.1000E 01	0.3411E 03	0.7303E 02	0.2515E 01
3	0.010	0.01	0.1003E 01	0.3411E 03	0.7303E 02	0.2515E 01
4	0.020	0.02	0.1008E 01	0.3411E 03	0.7303E 02	0.2515E 01
5	0.030	0.04	0.1015E 01	0.3411E 03	0.7303E 02	0.2515E 01
6	0.040	0.05	0.1024E 01	0.3411E 03	0.7303E 02	0.2516E 01
7	0.050	0.06	0.1033E 01	0.3411E 03	0.7303E 02	0.2517E 01
8	0.060	0.07	0.1043E 01	0.3411E 03	0.7304E 02	0.2518E 01
9	0.070	0.09	0.1055E 01	0.3411E 03	0.7304E 02	0.2519E 01
10	0.080	0.10	0.1066E 01	0.3411E 03	0.7305E 02	0.2521E 01
11	0.090	0.11	0.1078E 01	0.3412E 03	0.7306E 02	0.2523E 01
12	0.100	0.12	0.1091E 01	0.3412E 03	0.7307E 02	0.2526E 01
13	0.110	0.13	0.1103E 01	0.3412E 03	0.7309E 02	0.2529E 01
14	0.120	0.14	0.1116E 01	0.3412E 03	0.7310E 02	0.2532E 01
15	0.130	0.15	0.1129E 01	0.3412E 03	0.7312E 02	0.2536E 01
16	0.140	0.16	0.1143E 01	0.3412E 03	0.7314E 02	0.2540E 01
17	0.150	0.17	0.1156E 01	0.3412E 03	0.7316E 02	0.2545E 01
18	0.160	0.18	0.1169E 01	0.3412E 03	0.7318E 02	0.2550E 01
19	0.170	0.19	0.1182E 01	0.3412E 03	0.7320E 02	0.2555E 01
20	0.180	0.20	0.1195E 01	0.3413E 03	0.7323E 02	0.2561E 01
21	0.190	0.20	0.1207E 01	0.3413E 03	0.7326E 02	0.2567E 01
22	0.200	0.21	0.1220E 01	0.3413E 03	0.7329E 02	0.2574E 01
23	0.210	0.22	0.1232E 01	0.3413E 03	0.7332E 02	0.2581E 01
24	0.220	0.22	0.1244E 01	0.3413E 03	0.7335E 02	0.2588E 01
25	0.230	0.23	0.1255E 01	0.3414E 03	0.7338E 02	0.2595E 01
26	0.240	0.23	0.1266E 01	0.3414E 03	0.7342E 02	0.2603E 01
27	0.250	0.24	0.1276E 01	0.3414E 03	0.7346E 02	0.2611E 01

28	0.260	0.24	0.1286E 01	0.3414E 03	0.7350E 02	0.2619E 01
29	0.270	0.24	0.1295E 01	0.3414E 03	0.7354E 02	0.2628E 01
30	0.280	0.25	0.1303E 01	0.3415E 03	0.7358E 02	0.2637E 01
31	0.290	0.25	0.1311E 01	0.3415E 03	0.7362E 02	0.2646E 01
32	0.300	0.25	0.1318E 01	0.3415E 03	0.7367E 02	0.2655E 01
33	0.310	0.25	0.1324E 01	0.3416E 03	0.7371E 02	0.2664E 01
34	0.320	0.25	0.1329E 01	0.3416E 03	0.7376E 02	0.2673E 01
35	0.330	0.25	0.1333E 01	0.3416E 03	0.7380E 02	0.2682E 01
36	0.340	0.25	0.1336E 01	0.3416E 03	0.7385E 02	0.2691E 01
37	0.350	0.25	0.1338E 01	0.3417E 03	0.7390E 02	0.2701E 01
38	0.360	0.24	0.1339E 01	0.3417E 03	0.7394E 02	0.2710E 01
39	0.370	0.24	0.1339E 01	0.3417E 03	0.7399E 02	0.2719E 01
40	0.380	0.24	0.1338E 01	0.3418E 03	0.7404E 02	0.2728E 01
41	0.390	0.23	0.1336E 01	0.3418E 03	0.7409E 02	0.2737E 01
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59	0.570	0.07	0.1160E 01	0.3422E 03	0.7474E 02	0.2837E 01
60	0.580	0.06	0.1145E 01	0.3422E 03	0.7476E 02	0.2838E 01
61	0.590	0.05	0.1131E 01	0.3422E 03	0.7478E 02	0.2838E 01
62	0.600	0.04	0.1117E 01	0.3423E 03	0.7479E 02	0.2838E 01
63	0.610	0.02	0.1102E 01	0.3423E 03	0.7481E 02	0.2838E 01
64	0.620	0.01	0.1088E 01	0.3423E 03	0.7482E 02	0.2837E 01
65	0.630	-0.00	0.1074E 01	0.3423E 03	0.7483E 02	0.2836E 01
66	0.640	-0.01	0.1060E 01	0.3423E 03	0.7483E 02	0.2834E 01
67	0.650	-0.03	0.1046E 01	0.3423E 03	0.7484E 02	0.2832E 01
68	0.660	-0.04	0.1033E 01	0.3423E 03	0.7484E 02	0.2829E 01
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79	0.770	-0.16	0.9089E 00	0.3423E 03	0.7476E 02	0.2774E 01
80	0.780	-0.17	0.9001E 00	0.3423E 03	0.7474E 02	0.2767E 01
81	0.790	-0.19	0.8918E 00	0.3422E 03	0.7472E 02	0.2760E 01
82	0.800	-0.19	0.8838E 00	0.3422E 03	0.7471E 02	0.2753E 01
83	0.810	-0.20	0.8764E 00	0.3422E 03	0.7468E 02	0.2745E 01
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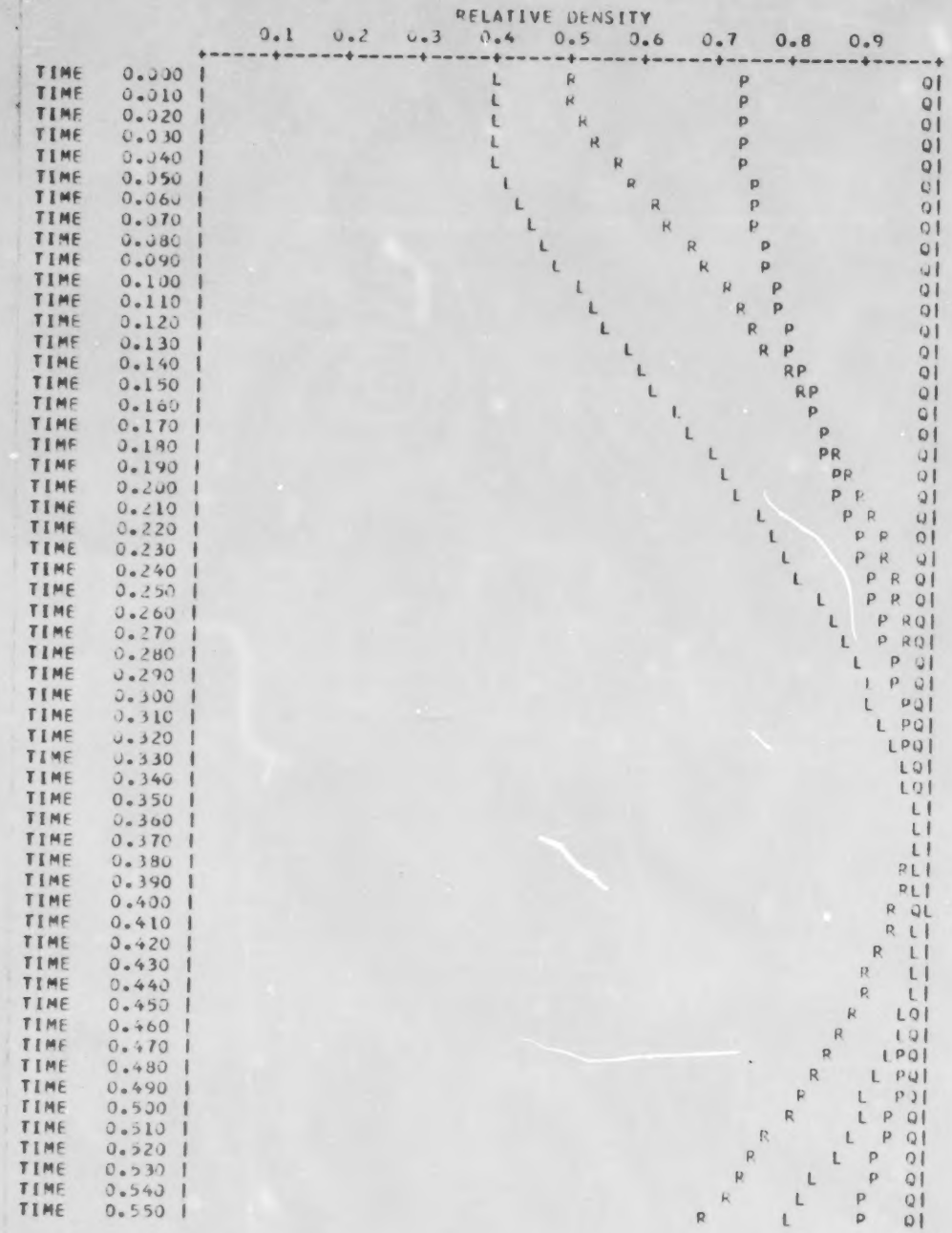
86	0.840	-0.22	0.8566E 00	0.3422E 03	0.7462E 02	0.2723E 01
87	0.850	-0.22	0.8509E 00	0.3422E 03	0.7459E 02	0.2715E 01

88	0.860	-0.23	0.8456E 00	0.3422E 03	0.7457E 02	0.2707E 01
89	0.870	-0.23	0.8408E 00	0.3421E 03	0.7454E 02	0.2699E 01
90	0.880	-0.24	0.8364E 00	0.3421E 03	0.7452E 02	0.2691E 01
91	0.890	-0.24	0.8324E 00	0.3421E 03	0.7449E 02	0.2682E 01
92	0.900	-0.24	0.8288E 00	0.3421E 03	0.7446E 02	0.2674E 01
93	0.910	-0.25	0.8257E 00	0.3421E 03	0.7444E 02	0.2666E 01
94	0.920	-0.25	0.8230E 00	0.3421E 03	0.7441E 02	0.2658E 01
95	0.930	-0.25	0.8207E 00	0.3421E 03	0.7438E 02	0.2650E 01
96	0.940	-0.25	0.8189E 00	0.3420E 03	0.7435E 02	0.2642E 01
97	0.950	-0.25	0.8174E 00	0.3420E 03	0.7432E 02	0.2634E 01
98	0.960	-0.25	0.8164E 00	0.3420E 03	0.7429E 02	0.2626E 01
99	0.970	-0.25	0.8158E 00	0.3420E 03	0.7426E 02	0.2618E 01
100	0.980	-0.25	0.8155E 00	0.3420E 03	0.7424E 02	0.2610E 01

********** END OF CALCULATION **********


RELATIVE DENSITY
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TIME 0.000
TIME 0.010
TIME 0.020
TIME 0.030
TIME 0.040
TIME 0.050
TIME 0.060
TIME 0.070
TIME 0.080
TIME 0.090
TIME 0.100
TIME 0.110
TIME 0.120
TIME 0.130
TIME 0.140
TIME 0.150
TIME 0.160
TIME 0.170
TIME 0.180
TIME 0.190
TIME 0.200
TIME 0.210
TIME 0.220
TIME 0.230
TIME 0.240
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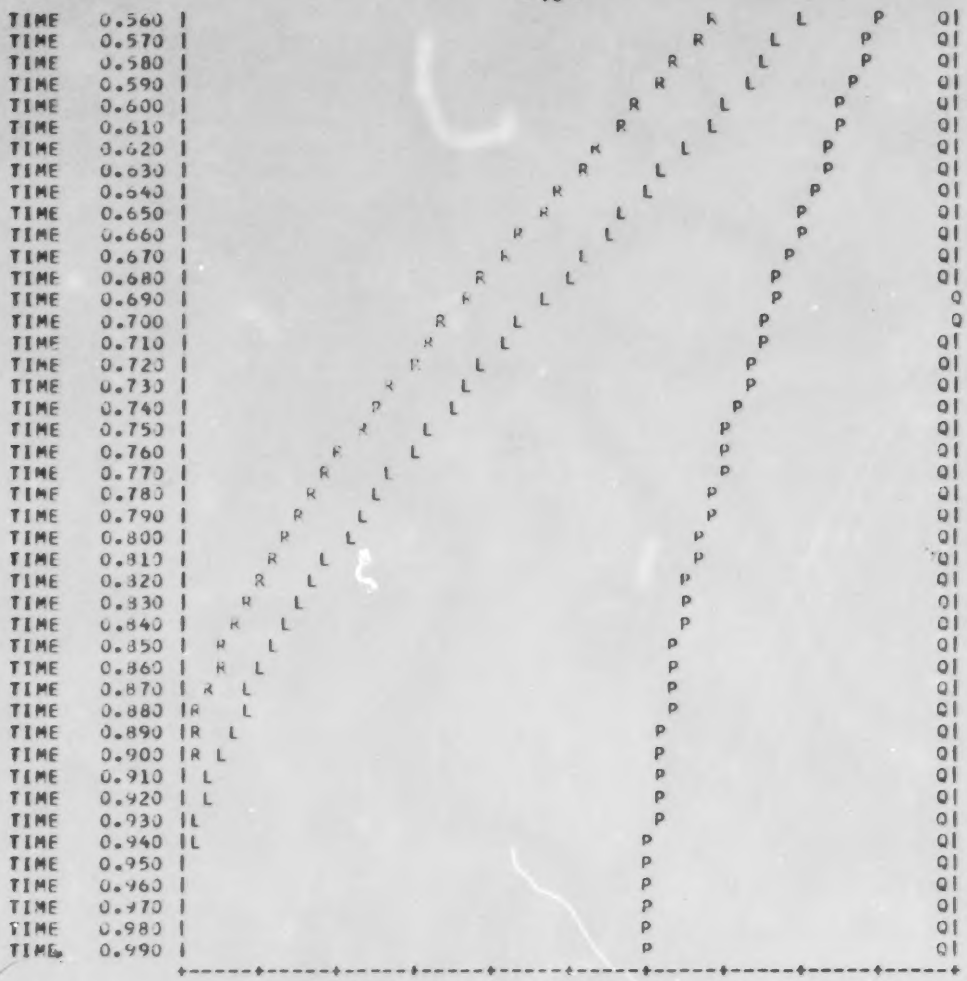
TIME 0.550



P - POWER
L - LOG. OF POWER
Q - PRECURSOR DENSITY
R - REACTIVITY

NEGATIVE VALUE PLOTTED WITH THE AXIS IN THE CENTER

REACTOR DYNAMICS MODULE, RD-2

REACTOR KINETICS WITH FEEDBACK

by

Ronald J. Onega

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Project Director: Milton C. Edlund

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REACTOR DYNAMICS MODULE, RD-2

REACTOR KINETICS WITH FEEDBACK

2.1 Object of Module

The object of this module is to:

(1) Examine the temperature feedback mechanism of a PWR and

(2) Solve the one delayed neutron model with temperature feedback for a step insertion and a ramp insertion of reactivity.

The time dependence of a reactor, taking the feedback mechanisms into account, is relatively difficult. We will consider a PWR core with a two path feedback. The reactivity is diminished as the temperature of the fuel increases due to the Doppler broadening of the resonances. This feedback is instantaneous since the temperature increase follows the power generated immediately. The second feedback path is that of the moderator temperature coefficient. As the moderator temperature increases, the number density decreases and the neutron mean free path increases so that leakage increases and reactivity decreases.

We will be concerned about the stability of the reactor to a limited degree. The dynamic response depends upon the magnitude of the temperature coefficients as well as that of the signs. For a given reactor design, i.e., a given lifetime and power level, the reactor may or may not be stable for a given set of reactivity coefficients.

The core region is the only one of interest in this module. The rest of the primary loop as well as the secondary loop is treated in an overall dynamics module for a PWR.

Also, all of our analysis will be fundamental mode analysis. The physical phenomena are taking place so slowly that the higher harmonics of the flux distribution are all dying out so rapidly that we only need to consider the lowest or fundamental mode.

The thermal analysis really should proceed by the solution of the space-time heat conduction equation. This is a very complicated procedure and would also mean that spatial effects of the kinetics equations should be taken into account. We, therefore, will assume only a lumped parameter model and will obtain the time dependence of a reactor which is really one with the average properties of the reactor under consideration.

The program name is FUMOTEM which is an acronym for "Fundamental Mode Kinetics with Temperature Feedback."

There are four types of reactivity inputs that the program can accommodate with NRO = 1, 2, 3 or 4 respectively:

1) $\rho_o(t) = \rho_o \qquad o \leq t \leq t_r$
$= o \qquad$ otherwise

2) $\rho_o(t) = \rho_o(1 + a\ t) \qquad o \leq t \leq t_r$
$= \rho_o(1 + a\ t_r) \qquad$ otherwise

3) $\rho_o(t) = \rho_o \cos a\ t$

or

4) $\rho_o(t) = \rho_o \sin a\ t$

The feedback reactivity is taken to be zero for times $t > t_r$. One can feed in a t_r greater than the calculation time however. So for $t > t_r$, the only reactivity is due to feedback effects.

2.2 The Feedback Model

We will use the one group delayed neutron model to describe the core neutronics. The kinetics equations are

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \lambda Q(t) \tag{2.2.1}$$

and

$$\frac{dQ(t)}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda Q(t) \tag{2.2.2}$$

where

$P(t)$ = The total reactor power (Megawatts)

$Q(t)$ = The power equivalence of the delayed neutron precursors (Megawatts).

Now we let ΔT_M be the deviation of the spatially averaged moderator temperature from its equilibrium value, i.e.,

$$\Delta T_M(t) = T_M(t) - T_{MO} \tag{2.2.3}$$

and similarly for the fuel temperature T_F we have

$$\Delta T_F(t) = T_F(t) - T_{FO} \tag{2.2.4}$$

where T_{MO} and T_{FO} are the equilibrium moderator and fuel temperatures respectively. Also, we let α_M and α_F be the moderator and fuel temperature coefficients of reactivity. Then (1)

$$\rho(t) = \rho_o(t) + \alpha_M \Delta T_M(t) + \alpha_F \Delta_F(t) \tag{2.2.5}$$

where generally, α_M and α_F will be negative or at least their sum is negative.

The temperature coefficient of reactivity for the moderator is

$$\alpha_M \equiv \frac{\partial \rho}{\partial T_M} \simeq \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_M} \tag{2.2.6}$$

and

$$\alpha_F \equiv \frac{\partial \rho}{\partial T_F} \simeq \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_F} \tag{2.2.7}$$

The thermal analysis of the core must now be considered and connected to the neutronics. The heat generated depends upon the fission rate or the power. We will look at the fuel temperature averaged over the core as well as an averaged coolant temperature. We will ignore the cladding of the fuel pins.

The heat balance equation for reactor fuel is

$$\begin{bmatrix}\text{Rate at which the}\\ \text{internal energy of}\\ \text{the fuel changes}\end{bmatrix} = \begin{bmatrix}\text{Production rate}\\ \text{of the energy}\\ \text{in the fuel}\end{bmatrix} - \begin{bmatrix}\text{Rate at which}\\ \text{heat is conducted}\\ \text{out of the fuel}\end{bmatrix}$$

or

$$\rho_F C_F V_F \frac{dT_F(t)}{dt} = P(t) - 4\pi k_F L_F (T_o - T_R) N \tag{2.2.8}$$

where

ρ_F = density of fuel (lb/ft^3)

C_F = specific heat of fuel (Btu/lb-°F)

V_F = volume of fuel (ft^3)

T_F = average temperature of the fuel (°F)

$P(t)$ = total power of reactor (Btu/hr)

k_F = thermal conductivity of fuel $(\frac{Btu}{hr\text{-}ft\text{-}^\circ F})$

L_F = length of fuel pin (ft)

T_o = centerline temperature of the fuel (°F)

T_R = temperature of the fuel pellets at the outer edge (pellet - water interface) (°F)

N = total number of fuel pins in the reactor.

The expression for Equation (2.2.8) was obtained from El-Wakil, "Nuclear Heat Transport", page 123, equation 5-48.

The temperatures are averaged over the fuel pins. If we number each of the pins in the core, then the centerline temperature is

$$T_0 = \frac{1}{N} (T_{01} + T_{02} + \ldots + T_{0N})$$

where T_{0i} is the centerline temperature for the ith pin. T_F is defined similarly so that it is the spatially averaged temperature of the "average fuel pin."

Equation (2.2.8) contains the centerline temperature and the temperature at the edge of the pellet T_R which must be eliminated. We assume that a parabolic temperature distribution holds even in the transient situation (really the transient heat conduction equation holds here) so that for one fuel pin we have

$$T(r) = T_o - \frac{P(t)r^2}{4k_F V_F} \qquad (2.2.9)$$

where T(r) is the temperature of the fuel pin a distance r from the center of the pin. The average fuel temperature is then

$$T_F(t) = \frac{2\pi}{\pi R_F^2} \int_0^{R_F} rdr \left[T_o - \frac{P(t) r^2}{4k_F V_F} \right]$$

$$= T_o - \frac{P(t)}{8k_F V_F} R_F^2 . \qquad (2.2.10)$$

Eliminating the centerline temperatures, we have, using (2.2.10),

$$T_F(t) = T_R(t) + \frac{R_F^2 P(t)}{8k_F V_F} . \qquad (2.2.11)$$

Equation (2.2.8) can now be written as

$$\rho_F C_F V_F \frac{dT_F(t)}{dt} = P(t) - 4\pi k_F L_F N \left[T_F(t) + \frac{P(t) R_F^2}{8k_F V_F} - T_R(t) \right]$$

$$= \left(1/2\right) P(t) - 4\pi k_F N L_F T_F(t) + 4\pi k_F L_F T_R(t) \qquad (2.2.12)$$

The wall temperature of the pellets $T_R(t)$ is connected to the coolant temperature $T_M(t)$ since

$$T_R(t) - T_M(t) = \frac{P(t)}{h_T A_F} \qquad (2.2.13)$$

where A_F is the total area of the fuel and h_T is the heat transfer coefficient for the fuel water interface (Btu/(Ft^2 hr-°F)).

The energy balance for the water in the core is

[Heat stored in water in core] = [Heat conducted in from fuel] - [Heat transferred out of reactor core]

or mathematically,

$$\rho_M C_M V_M \frac{dT_M(t)}{dt} = 2\pi R_F L_F h_T N[T_R(t) - T_M(t)] + \dot{m}_M[C_{M1}T_{M1} - C_{M2}T_{M2}] \qquad (2.2.14)$$

where $\dot{m}_M$ is the mass flow rate of the water through the core and $T_M(t)$ is the average moderator temperature. Also, assume $C_{M1} = C_{M2}$.

We will assume $\dot{m}_M$ is an input, as is T_{M1}, the moderator inlet temperature. The moderator outlet temperature is related to the inlet and average temperatures as

$$T_{M2}(t) = 2T_M(t) - T_{M1}$$

so that Equation (2.2.14) becomes (using Equation (2.2.13))

$$\rho_M C_M V_M \frac{dT_M(t)}{dt} = 2\pi R_F L_F h_T \cdot \left[\frac{P(t)}{h_T A_F}\right] + 2\dot{m}_M C_M [T_{M1} - T_M(t)]$$

$$= P(t) + 2\dot{m}_M C_M T_{M1} - 2\dot{m}_M C_M T_M(t). \qquad (2.2.15)$$

This can be rewritten as

$$\frac{dT_M(t)}{dt} = \frac{P(t)}{\rho_M C_M V_M} - \frac{2\dot{m}_M}{\rho_M V_M} \cdot T_M(t) + \frac{2\dot{m}_M}{\rho_M V_M} T_{M1} \qquad (2.2.16)$$

The area of the fuel is $2NL_F\pi R_F$ and the fuel volume is $\pi R_F^2 L_F N$.

In summary, the equations we must solve are Equations (2.2.1), (2.2.2), (2.2.12) and (2.2.16), the last two of which are written

$$\rho_F C_F V_F \frac{dT_F}{dt} = (1/2)P(t) - 4\pi k_F L_F N T_F(t) + 4\pi k_F L_F N T_M(t), \qquad (2.2.17)$$

and

$$\frac{dT_M(t)}{dt} = \frac{P(t)}{\rho_M C_M V_M} - \frac{2\dot{m}_M}{\rho_M V_M} T_M(t) + \frac{2}{\rho_M V_M} \dot{m}_M T_{M1}. \qquad (2.2.18)$$

If we put these equations into matrix form we have

$$\frac{d}{dt}\begin{bmatrix} P(t) \\ Q(t) \\ T_F(t) \\ T_M(t) \end{bmatrix} = \begin{bmatrix} \frac{\rho-\beta}{\Lambda} & \lambda & 0 & 0 \\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0 \\ \frac{1}{2\ \rho_F C_F V_F} & 0 & -\frac{4\pi K_F L_F N}{\rho_F C_F V_F} & \frac{4\pi K_F L_F N}{\rho_F C_F V_F} \\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & -\frac{2\dot{m}_M}{\rho_M V_M} \end{bmatrix} x$$

$$\begin{bmatrix} P(t) \\ Q(t) \\ T_F(t) \\ T_M(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{2}{\rho_M V_M} \dot{m}_M T_{M1}(t) \end{bmatrix}, \qquad (2.2.19)$$

or

$$\frac{d\ \underline{\phi}(t)}{dt} = \underline{\underline{A}}\ \underline{\phi} + \underline{B} \qquad (2.2.20)$$

where the $\underline{\underline{A}}$ is dependent upon the temperatures themselves. Thus, Equation (2.2.20) is non-linear.

The linearization of Equation (2.2.19) (or (2.2.20)) can be achieved assuming that we can look at changes about some operating point. Expanding about the operating point we have

$$P(t) = P^\circ + \Delta P \tag{2.2.21}$$

$$Q(t) = Q^\circ + \Delta Q \tag{2.2.22}$$

$$T_F(t) = T_F^\circ + \Delta T_F \tag{2.2.23}$$

$$T_M(t) = T_M^\circ + \Delta T_M \tag{2.2.24}$$

We assume that P° etc. are independent of time so

$$\frac{d\,\Delta\underline{\phi}}{dt} = (\underline{\underline{A}}^\circ + \Delta\underline{\underline{A}})\ (\underline{\phi}^\circ + \Delta\underline{\phi}) + \underline{B}$$

or neglecting the $\Delta\underline{\underline{A}} \cdot \Delta\underline{\phi}$ term we have

$$\frac{d}{dt}\Delta\underline{\phi} = \underline{\underline{A}}^\circ\ \underline{\phi}^\circ + \underline{\underline{A}}^\circ\ \Delta\underline{\phi} + \Delta\underline{\underline{A}}\ \underline{\phi}^\circ + \underline{B}$$

or

$$\frac{d}{dt}\Delta\underline{\phi} = \underline{\underline{A}}^\circ\ \Delta\underline{\phi} + \Delta\underline{\underline{A}}\underline{\phi}^\circ \tag{2.2.25}$$

since

$$\underline{\underline{A}}^\circ\ \underline{\phi}^\circ + \underline{B} = \underline{0}.$$

Writing Equation (2.2.25) out explicitly, we have

$$\frac{d}{dt}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix} = \begin{bmatrix}\frac{\rho_o-\beta}{\Lambda} & \lambda & 0 & 0\\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0\\ \frac{1}{2\rho_F C_F V_F} & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} & \frac{4\pi k_F L_F N}{\rho_F C_F V_F}\\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M}\end{bmatrix}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix}$$

$$+\begin{bmatrix}\frac{\alpha_F\Delta T_F + \alpha_M\Delta T_M}{\Lambda} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}P^\circ\\ Q^\circ\\ T_F^\circ\\ T_M^\circ\end{bmatrix}$$

or

$$\frac{d}{dt}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix} = \begin{bmatrix}\frac{\rho_o-\beta}{\Lambda} & \lambda & \frac{\alpha_F P^\circ}{\Lambda} & \frac{\alpha_M P^\circ}{\Lambda}\\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0\\ \frac{1}{2\rho_F C_F V_F} & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} & \frac{4\pi k_F L_F N}{\rho_F C_F V_F}\\ \frac{1}{\rho_M V_M C_M} & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M}\end{bmatrix}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix} \qquad (2.2.26)$$

Equation (2.2.26) is a linearized version of Equation (2.2.19). This equation is soluble by the ordinary method of eigenvalues. In matrix form, this can be written as

$$\frac{d\Delta \underline{\phi}}{dt} = \underline{\underline{A}}' \, \Delta\underline{\phi}, \qquad (2.2.27)$$

and $\underline{\underline{A}}'$ is given in terms of the equilibrium values.

The system of Equations (2.2.19) is highly non-linear because of the first element of the matrix $\underline{\underline{A}}$. The method discussed in Kinetics Module 1, developed by Hansen, is still applicable even to non-linear systems. There is a slight modification that we must discuss and this will be outlined in a later section.

The overall heat transfer coefficient is also relatively difficult to obtain so we discuss this and related parameters in the next section.

Problem 2.2.1

Show that if T_o is the temperature in the center of a fuel pin and T_M is the moderator temperature, then

$$T_o = T_M + \frac{P(t)a^2}{4k_F V_F} + \frac{P(t)}{2V_F}\left[\frac{1}{k_{clad}} \ln\frac{b}{a} + \frac{1}{h_T b}\right].$$

The inner radius of the clad is "a" and the outer radius is "b".

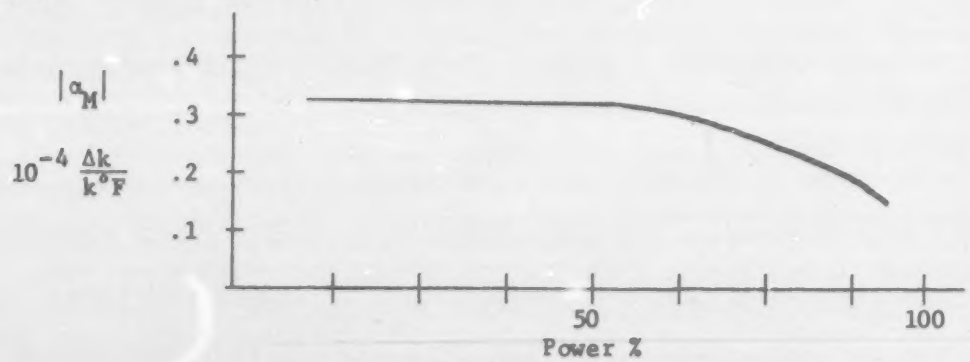
2.3 The Feedback Parameters

There are various parameters that must be obtained in order to numerically solve Equation (2.2.19). We list these parameters in order:

1. α_M - The moderator (coolant) temperature coefficient of reactivity.
2. α_F - The temperature coefficient of reactivity for the fuel.
3. C_{PF} - The total heat capacity of the fuel.
4. k_F - The thermal conductivity of the fuel.
5. h_T - The overall convective heat transfer coefficient of the fuel to moderator surface.

The flow rate F, the fuel volume V_F, etc. are all parameters that can be obtained from specifications of a particular reactor type and we shall not discuss these any further.

We first obtain a relation for α_M. The Final Safety Analysis Report of nuclear power plants generally have a curve of α_M as a function of power level for a given boron insertion and a critical rod insertion. The curves generally follow the pattern that α_M is constant from about 10% to 60% power and then decreases rather dramatically between 60 and 100% power. α_M is assumed to be negative.



We will therefore take α_M to be an input constant but an accurate analysis would necessitate a knowledge of the variation of α_M with power.

The $\alpha_F(P)$ is also rather difficult to calculate. To do this, we use Equation (2.2.7), i.e.

$$\alpha_F \simeq \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_F} . \tag{2.3.1}$$

Now if we use the relation

$$k_{eff} = \eta p f \varepsilon L_f L_{th} \tag{2.3.2}$$

then

$$\ell n(k_{eff}) = \ell n(\eta f \varepsilon L_f L_{th}) + \ell n(p)$$

and if we assume that the resonance escape probability p is the only factor which changes with the fuel temperature, then

$$\alpha_F = \frac{1}{k_{eff}} \frac{dk_{eff}}{dT_F} = \frac{1}{p} \frac{dp}{dT_F} . \tag{2.3.3}$$

A standard expression for the resonance escape probability is (2)

$$p = e^{-\frac{N_F V_F}{\xi_M \Sigma_M V_M} R(T_F)} \tag{2.3.4}$$

and the resonance integral $R(T_F)$ is given by the empirical relation

$$R(T_F) = R(T_{FO})[1 + \gamma(\sqrt{T_F} - \sqrt{T_{FO}})] \tag{2.3.5}$$

where T_{FO} is the equilibrium temperature of the fuel.

We now relate "p" to α_F in a way so that the α_F can be calculated. Let

$$a_5 \equiv \frac{N_F V_F}{\xi_M \Sigma_{SM} V_M} \tag{2.3.6}$$

then

$$\ell n\ p = - a_5 R(T_F), \tag{2.3.7}$$

and

$$\alpha_F = - a_5 \frac{dR(T_F)}{dT_F} = - a_5 \frac{R(T_{FO})\gamma}{2\sqrt{T_F}}. \tag{2.3.8}$$

IF we substitute T_{FO} into p we get

$$p(T_{FO}) = e^{-a_5 R(T_{FO})}$$

or

$$\ell n \frac{1}{p(T_{FO})} = a_5 R(T_{FO}). \tag{2.3.9}$$

For an arbitrary value of temperature T_F we obtain

$$\ell n \frac{1}{p(T_F)} = a_5 R(T_F). \tag{2.3.10}$$

Dividing Equation (2.3.9) by Equation (2.3.10) we get

$$\frac{R(T_{FO})}{R(T_F)} = \frac{\ell n \frac{1}{p(T_{FO})}}{\ell n \frac{1}{p(T_F)}} = + \frac{\ell n\ p(T_{FO})}{\ell n\ p(T_F)}. \tag{2.3.11}$$

Inserting Equation (2.3.11) into Equation (2.3.8) we obtain

$$\alpha_F = -\frac{\gamma}{2\sqrt{T_F}}\left[a_5\, R(T_{FO})\right] = -\frac{\gamma a_5}{2\sqrt{T_F}}\, R(T_F) \cdot \frac{\ell n\ p\ (T_{FO})}{\ell n\ p\ (T_F)} .$$

Equation (2.3.10) yields

$$a_5\, R(T_F) = -\ \ell n\ p(T_F)$$

so

$$\alpha_F = -\frac{\gamma}{2\sqrt{T_F}} \cdot \left[-\ \ell n\ p(T_F)\right] \cdot \frac{\ell n\ p(T_{FO})}{\ell n\ p(T_F)}$$

$$= -\frac{\gamma}{2\sqrt{T_F}}\ \frac{1}{\ell n\ p(T_{FO})} . \qquad (2.3.12)$$

In our module, we will assume that γ and $p(T_{FO})$ are read in.

The third item of discussion is C_{PF}, the total heat capacity of the fuel. It is obvious that

$$C_{PF} = V_F \cdot C'_{PF} \cdot \rho_F \qquad (2.3.13)$$

where C'_{PF} is the specific heat capacity of the fuel, i.e. the units are Btu/lbm-°F, ρ_F is the density of fuel and V_F is the volume of fuel. There is a small variation of C_{PF} with temperature but it is small for UO_2 in the region of interest. The value that will be of interest for us is 0.0590 Btu/lbm-°F.

The thermal conductivity of the fuel is a function of the temperature. El Wakil (3) lists values of k_F as a function of T_F. We use these values to obtain a polynomial regression of the k_F with the T_F.

The last quantity that we discuss is the overall convective heat transfer coefficient h_T. The heat transfer coefficient is defined by Newton's law of cooling. The relevant relation is

$$\frac{P(t)}{A_F} = h_T(T_F - T_M), \tag{2.3.14}$$

where A_F is the fuel area. There are many factors which influence h_T such as

i) the temperature of the system

ii) the heat flux

iii) the physical properties of the moderating material

iv) the geometrical shape of the cooling surface

v) the flow rate of the coolant

In the PWR, the coolant flow is turbulent. Therefore, to obtain the heat transfer coefficient h_T we assume that

$$h_T = \frac{Ck_M Re^{0.8} Pr^{0.333}}{D_e} \tag{2.3.15}$$

where

D_e = The equivalent diameter of flow channels through the fuel rod bundles

k_M = The thermal conductivity of the moderator

C = The Colburn correction factor for fluid flow parallel to the tube bundles

Re = Reynolds number

Pr = Prandtl number of the coolant in the core

Equation (2.3.15) was a correlation recommended by Weisman. The Prandtl number is taken from the 1967 ASME Steam Tables.

The equivalent diameter of the flow channel is shown in Fig. 2.3.2. If we let "a" be the area of the flow channel and P_w be the wetted perimeter, then

$$D_e = \frac{4A}{P_w} . \tag{2.3.16}$$

For a typical PWR, D, the diameter of the fuel rods is 0.03583 ft and the pitch (the distance between centerlines of fuel rods) is designated as P_c. The Colburn correction factor is an empirical relation and is

$$C = 0.042 \frac{P_c}{D} - 0.024. \tag{2.3.17}$$

So we denote the heat transfer coefficient, which varies with power and therefore with time, as $h_T(t)$ and the equilibrium value as h(0). Using Equation (2.3.15) we have

$$\frac{h_T(t)}{h_T(0)} = \frac{k_M(t)\ Re^{0.8} Pr^{1/3}}{k_M(0)\ Re^{0.8}(0)\ Pr^{1/3}(0)} ,$$

or

$$h_T(t) = \frac{h(0)\ k_M(t)}{k_M(0)\ Pr^{1/3}(0)} \left(\frac{\rho_\mu(t)}{\mu(t)}\right)^{0.8} \frac{Pr(t)^{1/3}}{\left(\frac{\rho_M(0)}{\mu(0)}\right)^{0.8}} , \tag{2.3.18}$$

where we have used the fact that

$$Re = \frac{\rho_M\ vD_e}{\mu} , \tag{2.3.19}$$

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$$\mu_M(T) = 8.5067\times10^{-1} - 1.7501\times10^{-3}\, T_M + 1.1142\times10^{-6}\, T_M^2 \qquad (2.3.23)$$

and for the thermal conductivity of the fuel, we have

$$k_F(T) = 6.102141 - 4.636522\times10^{-3}\, T_F + 1.306299\times10^{-6}\, T_F^2. \qquad (2.3.24)$$

The temperature is for the fuel in the case of k_F but the moderator temperature otherwise.

Problem 2.3.1

Given the following data:

R_F = 0.015042 ft

Fuel length = 12.00 ft

Volume of reactor vessel = 3643 ft^3

Number of fuel assemblies = 145

Fuel rods per assembly = 208

Rod pitch = 0.04733 ft

ρ = 43.214 lbm/ft^3

v = 16.3 ft/sec

$\mu = 5.828 \times 10^{-5}$

D_e = 0.04377 ft

Re = 528,998

Pr = 1.01

k_M = 0.3010 Btu/hr-ft-°F

a) show the Colburn correction is 0.03148

b) h = 8232 Btu/ft^2-hr-°F.

2.4 Numerical Solution of the Dynamics Equations

The numerical integration of the differential equations can be carried out once the various parameters, such as the heat transfer coefficient, thermal conductivity, etc., have been obtained. We again write the equations so that we can proceed to use Hansen's method in their solution. FUMOTEM solves the kinetics equations using Hansen's method. The system of equations is

$$\frac{d\underline{\phi}(t)}{dt} = \underline{\underline{A}}\,\underline{\phi}(t) + \underline{B} \tag{2.4.1}$$

where

$$\underline{\phi} = \begin{bmatrix} P(t) \\ Q(t) \\ T_F(t) \\ T_M(t) \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{2\,\dot{m}_M T_{M1}(t)}{\rho_M V_M} \end{bmatrix} \tag{2.4.2}$$

and

$$\underline{\underline{A}} = \begin{bmatrix} \dfrac{\rho(t)-\beta}{\Lambda} & \lambda & 0 & 0 \\ \dfrac{\beta}{\Lambda} & -\lambda & 0 & 0 \\ \dfrac{1}{2\rho_F C_F V_F} & 0 & \dfrac{-4\pi k_F L_F N}{\rho_F C_F V_F} & \dfrac{4\pi k_F L_F N}{\rho_F C_F V_F} \\ \dfrac{1}{\rho_M C_M V_M} & 0 & 0 & \dfrac{-2\,\dot{m}_M}{\rho_M V_M} \end{bmatrix}, \tag{2.4.3}$$

subject to the constraint that

$$\rho(t) = \rho_o + \alpha_M \Delta T_M + \alpha_F \Delta T_F. \qquad (2.4.4)$$

All the parameters involved in Equations (2.4.1) - (2.4.4) are defined in section 2.2.

Hansen's method must be generalized slightly for the solution of Equation (2.4.1). We break the $\underline{\underline{A}}$ matrix into three parts:

$$\underline{\underline{A}} = \underline{\underline{L}} + \underline{\underline{D}} + \underline{\underline{U}}, \qquad (2.4.5)$$

with

$$\underline{\underline{D}}(t) = \begin{bmatrix} \frac{\rho(t)-\beta}{\Lambda} & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} & 0 \\ 0 & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M} \end{bmatrix},$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{\beta}{\Lambda} & 0 & 0 & 0 \\ \frac{1}{2\rho_F C_F V_F} & 0 & 0 & 0 \\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & 0 \end{bmatrix}$$

and

$$\underline{\underline{U}} = \begin{bmatrix} 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{4\pi k_F L_F N}{\rho_F C_F V_F} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation (2.4.1) can then be written as

$$\frac{d\underline{\phi}}{dt} - \underline{\underline{D}}(t)\,\underline{\phi}(t) = (\underline{\underline{L}} + \underline{\underline{U}})\,\underline{\phi}\,(t) + \underline{B} \quad . \tag{2.4.6}$$

We wish to develop an iteration procedure for the solution of this system so we begin the calculation at time t_0 and advance to time t_1, and define

$$h = t_1 - t_0 \quad . \tag{2.4.7}$$

Before we use this relation though, let's multiply each term of Equation (2.4.6) by the integrating factor $e^{-\int_o^t \underline{\underline{D}}(t')dt'}$ so

$$e^{-\int_o^t \underline{\underline{D}}(t')dt'} \frac{d\underline{\phi}}{dt} - e^{-\int_o^t \underline{\underline{D}}(t')dt'} \underline{\underline{D}}(t)\phi(t) = e^{-\int_o^t \underline{\underline{D}}(t')dt'} (\underline{\underline{L}}+\underline{\underline{U}})\phi(t) + e^{-\int_o^t \underline{\underline{D}}(t')dt'} \underline{B} \quad .$$

or

$$\frac{d}{dt}\{e^{-\int_o^t \underline{\underline{D}}(t')dt'} \underline{\phi}(t)\} = e^{-\int_o^t \underline{\underline{D}}(t')dt'} (\underline{\underline{L}} + \underline{\underline{U}})\phi(t) + e^{-\int_o^t \underline{\underline{D}}(t')dt'} \underline{B}. \tag{2.4.8}$$

The left side is an exact differential so we integrate from 0 to h and obtain

$$e^{-\int_o^t \underline{D}(t')dt'} \underline{\phi}(t) \Big|_0^h = \int_0^h e^{-\int_o^t \underline{D}(t')dt'} (\underline{L}+\underline{U})\underline{\phi}(t)dt + \int_0^h e^{-\int_o^t \underline{D}(t')dt'} \underline{B}(t)dt$$

or

$$\underline{\phi}(t_o + h) = e^{+\int_o^h \underline{D}(t')dt'} \underline{\phi}(t_o) + \int_o^h e^{\int_o^h \underline{D}(t')dt'} e^{-\int_o^t \underline{D}(t')dt'} (\underline{L} + \underline{U})\underline{\phi}(t_o+\theta)d\theta$$

$$+\int_o^h e^{+\int_o^h \underline{D}(t')dt' - \int_o^t \underline{D}(t')dt'} \underline{B}(t_o + \theta)d\theta \ . \qquad (2.4.9)$$

Here

$$t_o < \theta < t_1 = t_o + h$$

and

$$d\theta = dt.$$

Notice in Equation (2.4.9) that

$$e^{\int_o^h \underline{D}(t')dt' - \int_o^t \underline{D}(t')dt'} = e^{\int_t^h \underline{D}(t')dt'} = e^{\int_{t_o+\theta}^h \underline{D}(t')dt'} .$$

Therefore the equation that must be solved is

$$\underline{\phi}(t_o + h) = e^{\int_o^h \underline{D}(t')dt'} \phi(t_o) + \int_o^h e^{\int_{t_o+\theta}^h \underline{D}(t')dt'} (\underline{L} + \underline{U})\Big|_{t_o+\theta} \underline{\phi}(t_o + \theta)dt$$

$$+\int_{o}^{h} e^{\int_{t_o+\theta}^{h} \underline{D}(t')dt'} \underline{B}(t_o + \theta)d\theta \quad . \tag{2.4.10}$$

Equation (2.4.10) is an integral equation so we must approximate its solution. To do this, we assume that

$$\underline{\phi}(t_o + \theta) = e^{\omega_o \theta} \underline{\phi}(t_o) \tag{2.4.11}$$

where ω_o is the largest eigenvalue of the matrix $\underline{\underline{A}}$ evaluated at time t_o. This means we must solve the equation

$$\left|\underline{\underline{A}} - \omega\underline{\underline{I}}\right| = 0$$

or

$$\det \begin{bmatrix} \frac{\rho(t_o)-\beta}{\Lambda} - \omega & \lambda & 0 & 0 \\ \frac{\beta}{\Lambda} & -\lambda-\omega & 0 & 0 \\ \frac{1}{2\rho_F C_F V} & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} - \omega & \frac{4\pi k_F L_F N}{\rho_F C_F V_F} \\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & \frac{-2\pi_M}{\rho_M V_M} - \omega \end{bmatrix} = 0, \tag{2.4.12}$$

to obtain the eigenvalues, ω_o, ω_1, ω_2 and ω_3, where

$$\omega_o > \omega_1 > \omega_2 > \omega_3 .$$

The ω's are, of course, time dependent quantities.

We also look at $\underline{B}(t_o + \theta)$ and assume that we can expand it in a Taylor series so that

$$\underline{B}(t_o + \theta) = \underline{B}(t_o) + \theta \frac{d\underline{B}}{dt}(t_o) + \frac{1}{2}\theta^2 \frac{d^2\underline{B}(t_o)}{d\theta^2} + \ldots \qquad (2.4.13)$$

and also

$$\underline{\underline{L}}(t_o + \theta) + \underline{\underline{U}}(t_o + \theta) \simeq \underline{\underline{L}}(t_o) + \underline{\underline{U}}(t_o) + \ldots \qquad (2.4.14)$$

We shall keep only the 1st terms of these expansions.

Inserting Equations (2.4.14), (2.4.13) and (2.4.11) into Equation (2.4.10) we get (after assuming that $\int_o^h \underline{\underline{D}}(t')dt' \simeq \underline{\underline{D}}(t_o)h$)

$$\underline{\phi}(t_o + h) = e^{\underline{\underline{D}}h}\underline{\phi}(t_o) + \int_o^h e^{\underline{\underline{D}}(h-\theta)}(\underline{\underline{L}} + \underline{\underline{U}})\, e^{\omega_o\theta}\underline{\phi}(t_o)\, d\theta$$

$$+ \int_o^h e^{\underline{\underline{D}}(h-\theta)}\underline{B}(t_o)d\theta$$

$$= e^{\underline{\underline{D}}h}\underline{\phi}(t_o) + e^{\underline{\underline{D}}h}(\omega_o\underline{\underline{I}} - \underline{\underline{D}})^{-1}\left[e^{(\omega_o\underline{\underline{I}} - \underline{\underline{D}})\theta}\right]_o^h (\underline{\underline{L}} + \underline{\underline{U}})\underline{\phi}(t_o)$$

$$+ e^{\underline{\underline{D}}h}(-\underline{\underline{D}}^{-1})\, e^{-\underline{\underline{D}}\theta}\Big|_o^h \underline{B}(t_o).$$

Putting the limits into this equation and simplifying we finally have

$$\underline{\phi}(t_o + h) = e^{\underline{D}h}\underline{\phi}(t_o) + (\omega_o\underline{I} - \underline{D})^{-1}[e^{\omega_o h\underline{I}} - e^{\underline{D}h}][\underline{L}(t_o) + \underline{U}(t_o)]\underline{\phi}(t_o)$$

$$+ \underline{D}^{-1}(e^{\underline{D}h} - \underline{I})\underline{B}(t_o) \quad . \tag{2.4.15}$$

If we now set

$$\underline{\phi}(t_j) = \underline{\phi}_j$$

and

$$\underline{\phi}(t_j + h) = \underline{\phi}_{j+1} \quad .$$

Then Equation (2.4.15) becomes

$$\underline{\phi}_{j+1} = \underline{H}\,\underline{\phi}_j + \underline{R}\,\underline{B}_j \tag{2.4.16}$$

where

$$\underline{H} \equiv e^{\underline{D}h} + (\omega_o\underline{I} - \underline{D})^{-1}[e^{\omega_o\underline{I}h} - e^{\underline{D}h}](\underline{L} + \underline{U}) \tag{2.4.17}$$

with all quantities evaluated at time t_o and

$$\underline{R} = \underline{D}^{-1}(e^{\underline{D}h} - \underline{I}) \quad . \tag{2.4.18}$$

We now write $\underline{H}$ and $\underline{R}$ explicitly for our problem. To do this, consider Equation (2.4.17) and the explicit relation for $\underline{L}$ and $\underline{U}$; then

$$\underline{H} = \begin{bmatrix} H_1 & H_2 & 0 & 0 \\ H_3 & H_4 & 0 & 0 \\ H_5 & 0 & H_6 & H_7 \\ H_8 & 0 & 0 & H_9 \end{bmatrix}, \tag{2.4.19}$$

where

$$H_1 = e^{\left[\frac{\rho(t_o)-\beta}{\Lambda}\right]h} ,$$

$$H_2 = \lambda\left[\frac{e^{\omega_o h} - e^{\frac{\rho(t_o)-\beta}{\Lambda}h}}{\omega_o - \frac{\rho(t_o)-\beta}{\Lambda}}\right] ,$$

$$H_3 = \frac{\beta}{\Lambda}\left[\frac{e^{\omega_o h} - e^{-\lambda h}}{\omega_o + \lambda}\right] ,$$

$$H_4 = e^{-\lambda h} ,$$

$$H_5 = \frac{1}{2\rho_F C_F V_F}\left[\frac{e^{\omega_o h} - e^{\frac{4\pi k_F L_F N}{\rho_F C_F V_F}h}}{\omega_o - \frac{4\pi k_F L_F N}{\rho_F C_F V_F}}\right] ,$$

$$H_6 = e^{-\frac{4\pi k_F L_F N}{\rho_F C_F V_F}h} ,$$

$$H_7 = \frac{4\pi k_F L_F N}{\rho_F C_F V_F}\left[\frac{e^{\omega_o h} - e^{\frac{4\pi k_F L_F N}{\rho_F C_F V_F}h}}{\omega_o - \frac{4\pi k_F L_F N}{\rho_F C_F V_F}}\right] ,$$

$$H_8 = \frac{1}{\rho_M C_M V_M}\left[\frac{e^{\omega_o h} - e^{-\frac{2\dot{m}_M h}{\rho_M V_M}}}{\omega_o + \frac{2\dot{m}_M}{\rho_M V_M}}\right],$$

and

$$H_9 = -\frac{2\dot{m}_M}{\rho_M V_M}\left[\frac{e^{\omega_o h} - e^{-\frac{2\dot{m}_M}{\rho_M V_M} h}}{\omega_o + \frac{2\dot{m}_M}{\rho_M V_M}}\right],$$

and the $\underline{\underline{R}}$ matrix is

$$\underline{\underline{R}} = \begin{bmatrix} \frac{\Lambda}{\rho-\beta}\left(e^{\frac{\rho-\beta}{\Lambda}}-1\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\lambda}(1-e^{-\lambda h}) & 0 & 0 \\ 0 & 0 & \frac{\rho_F C_F V_F}{4\pi k_F L_F N}\left(1-e^{-\frac{4\pi k_F L_F N h}{\rho_F C_F V_F}}\right) & 0 \\ 0 & 0 & 0 & \frac{\rho_M V_M}{2\dot{m}_M}\left(1-e^{\frac{-2\dot{m}_M}{\rho_M V_M} h}\right) \end{bmatrix} \qquad (2.4.20)$$

The basic iteration procedure in FUMOTEM is as follows:

1. Calculate the pertinent reactor parameters such as α_F, h_T, $\dot{m}$, ρ_M, μ, C_M, k_F, and k_M for the guessed initial conditions. The parameters read in are T_{M1}, α_M, h, P_o, v (the coolant velocity), some optional settings for print out and the time length the program is to operate.

2. Construct the vector $\underline{\phi}(0)$. We can choose the parameters of interest, i.e.

$$P(0) = 500 \text{ MW},$$

and

$$Q(0) = \frac{\beta}{\Lambda\lambda} P(0) \text{ MW}.$$

The fuel temperature $T_F(0)$ and the moderator temperature $T_M(0)$ are completely determined by P(0) and the above read-in parameters. $T_F(0)$ and $T_M(0)$ are difficult to obtain since C_M, k_M, μ etc. depend on the temperature. This difficulty is overcome by using an iteration procedure. The method is as follows:

a) From P(0) and initially guessed values $T_F^{(0)}(0)$ and $T_M^{(0)}(0)$, calculate $\dot{m}_M$ from the relation

$$\dot{m}_M(T_M) = \rho_M(T_M) \text{ Av}. \qquad (2.4.21)$$

Equation (2.3.20) is used to obtain $\rho_M(T_M)$ from the guessed moderator temperature $T_M^{(0)}(0)$. Equation (2.2.15) is now used to obtain an improved guessed power $P^{(1)}(0)$,

$$P^{(1)}(0) = 2\dot{m}_M C_M (T_M^{(0)}(0) - T_{M1}). \qquad (2.4.22)$$

b) The fractional difference between the actual power and $P^{(1)}(0)$ is

$$E \equiv \left| \frac{P^{(1)}(0) - P(0)}{P(0)} \right| \qquad (2.4.23)$$

If $E \leq \Delta$ ($\Delta = 0.01$ and is an input) then the guessed temperature $T_M^{(0)}(0)$ is the correct moderator temperature. If

$$E > \Delta \qquad (2.4.24)$$

then change $T_M^{(0)}(0)$ to $T_M^{(1)}(0)$ and repeat the above until convergence is achieved.

c) The $T_M(0)$ is now used in Equation (2.2.17) which, for equilibrium, becomes, after solving for $T_F(0)$,

$$T_F(0) = T_M(0) + \frac{1}{8\pi k_F L_F N} P(0). \qquad (2.4.25)$$

This step also requires an iterative procedure since k_F is a function of the fuel temperature as seen from Equation (2.4.25).

3. Determine the largest eigenvalue of the equation

$$|\underline{\underline{A}} - \omega \underline{\underline{I}}| = 0.$$

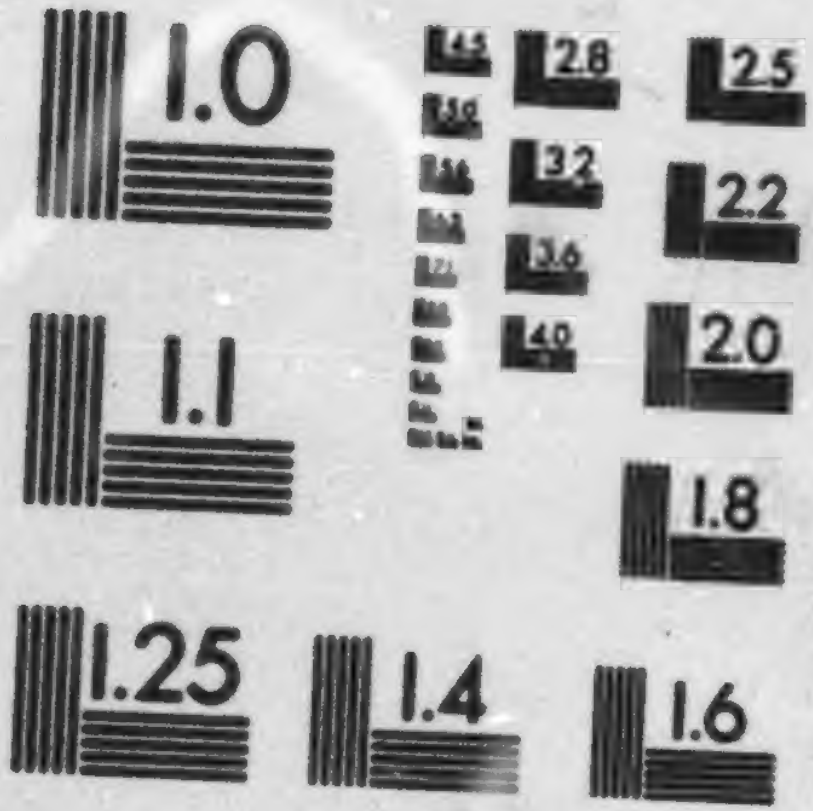
This will be the solution of a 4 x 4 determinant which is rather easy on the computer. From this determine the largest root ω_o. The Newton-Raphson method is used to calculate all four roots of this polynomial and then the largest root is picked by comparison of the roots. The subroutine POLRT is used for the determination of ω_o for each time step. Module RD-1 describes the Newton-Raphson method.

4. Construct the $\underline{\underline{H}}(t_o)$ matrix using Equation (2.4.19) and $\underline{\underline{R}}(t_o)$ using (2.4.20)

5. Determine the vector $\underline{\phi}_1$ from Equation (2.4.16), i.e.

$$\underline{\phi}_1 = \underline{\underline{H}}(t_o)\underline{\phi}_o + \underline{\underline{R}}(t_o)\underline{B}_o.$$

6. Repeat the above procedure using step 3 and continue as long as required to achieve the solution over the time domain of interest. The time steps are chosen by the criterion $h = \left|\frac{1}{\omega_o}\right|$ and h is constrained to be in the interval $0.005 \leq h \leq 0.05$ sec.



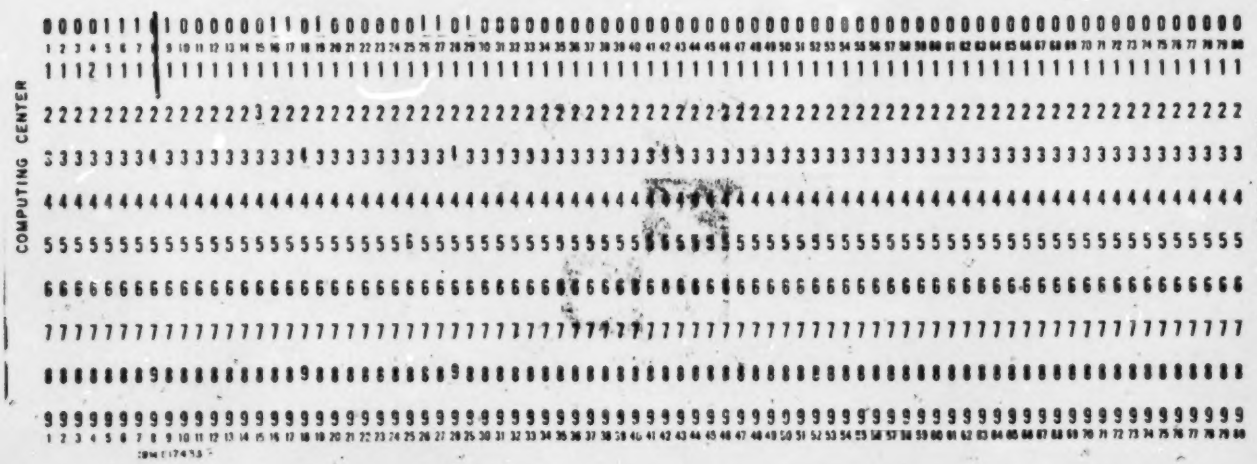
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

2.5 Input-Output Data for Code FUMOTEM

The input data required for the program are presented below:

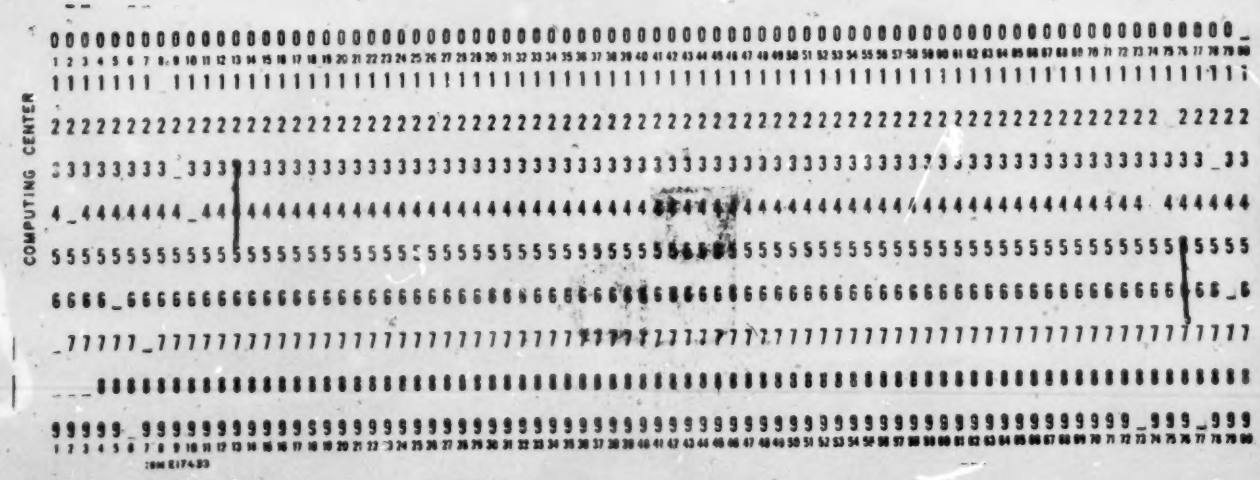
Data Card Number	Format Statement Number	Format	Unit	Variable Name	Description
1	20	I2	--	NOPLT	Plotting option 1 -- plot 0 -- no plot
2	30	F10.5	--	BETA	Delayed neutron fraction
		F10.5	sec^{-1}	X	Delayed neutron decay constant
		F10.7	sec^{-1}	XL	Neutron generation time
		F10.7	$	RO	reactivity, ρ_o
		I10	--	NRO	option for the type of reactivity insertion
		F10.5	sec	RTIME	Time duration reactivity is inserted for ramp input (NRO=2)
		F10.5	sec^{-1}	A	reactivity insertion rate
3	40	F10.6	$°F^{-1}$	AM	Moderator temperature coefficient of reactivity
		F10.6	$(°F)^{\frac{1}{2}}$	G	Constant for resonance capture
		F10.6	--	PTFO	Resonance escape probability, P_o
4	50	F10.5	ft	RF	Fuel radius, R_F
		F10.5	ft	PC	Distance between fuel pin centers, P
		F10.5	$\frac{Btu}{lb-°F}$	CPFX	Specific heat of the fuel, C_F
		F10.2	lb/ft^3	FDENS	Fuel density, ρ_F
		F10.2	ft	FH	Fuel height, L
		I10	--	NA	Number of fuel assemblies
		I10	--	NFRPA	Number of fuel rods per assembly
5	60	F10.2	ft/sec	V	Average velocity of coolant, v
		F10.2	°F	TMI	Inlet coolant temperature to the core T_{M1}.
6	70	F10.4	sec	TE	End of calculation time
		F10.4	sec	DH	Time increment
		F10.4	--	DELTA	Convergence criterion for the calculation of the initial equilibrium state
7	80	F10.2	MW	PPW	Initial equilibrium power
		F10.2	°F	TGUES	Guessed initial moderator temperature to obtain equilibrium
		F10.2	°F	TGF	Guessed initial fuel temperature for obtaining equlibrum conditions.

```
1                                                                          (1)
  0.00645   0.07695 0.0001  +0.50              2      0.5   2.5           (2)
-0.0001     0.00002 0.80                                                   (3)
  0.01504   0.04733 0.0590      43.2    18.0        145       208         (4)
   13.0     400.0                                                         (5)
   8.0      0.01    0.01                                                  (6)
  1000.0    200.0   500.0                                                 (7)
```



In addition to these seven data cards, there is an eighth card which has the impression as shown below. This card is necessary for the plot routine.

```
1 FRP1LM                                                        RD2 9361
```



The output of FUMOTEM consists of three parts. The first is simply a writing of all the input data. The second is the equilibrium state calculation and the third block of data is the fuel temperature, moderator temperature reactivity, exit temperature, power, α_F and Q as a function of time. If the user specifies, a plot of power, precursor power, T_F, T_M and reactivity as a function of time is provided.

Problem 2.5.1

Run FUMOTEM for the sample data cards shown.

Problem 2.5.2

Run FUMOTEM for the reactivity input

$$\rho(t) = \$0.50 \cos 2.5t$$

with the following parameters:

NOPLT = 1	α_M = - 0.00005
β = 0.00645	
λ = 0.07695 sec^{-1}	γ = 0.0002
RTIME = 4.0 sec	P_o = 0.80
R_F = 0.01504 ft	Number of fuel assemblies = 745
P_C = 0.04733 ft	
C_{PF} = 0.0590 Btu/lb-°F	Number of fuel pins per assembly = 208
ρ_F = 43.2 lb/ft^3	
L = 12 ft	T_{M1} = 400°F
TE = 4.0 sec	DELTA = 0.01

DH = 0.01 sec

P_o = 1000 MW

TGUES = 200.0°F

TGF = 500.0°F

v = 13.0 ft/sec

Also run the same calculation for v = 26.0 ft/sec.

FUMOTEM is written in FORTRAN in single precision except for the eigenvalue calculation. The solution of the equation

$$|\underline{\underline{A}} - \omega \underline{\underline{I}}| = 0$$

for the root ω_0, ω_1, ω_2, ω_3 is done in double precision. All four roots are determined and the largest one is picked to form the $\underline{\underline{H}}$ and $\underline{\underline{R}}$ matrices.

The memory requirement is about 40 kilobytes and the execution time varies with the time length. The reactor is to be simulated. Generally it takes about 1 1/2 seconds of computer time to simulate one second of reactor transient time.

REFERENCES

1. D. L. Hetrick, "Dynamics of Nuclear Reactors," The University of Chicago Press, 1971, page 159.

2. J. R. Lamarsh, "Nuclear Reactor Theory," Addison-Wesley Publishing Company, Reading, Massachusetts, 1966, pages 218, 459.

3. M. M. El-Wakil, "Nuclear Heat Transport," International Textbook Company, Scranton, Pa., 1971, page 104.

List of Symbols for FUMOTEM

The following symbols are listed in alphabetical order in the FUMOTEM program.

A	a	Period of reactivity insertion
AF	α_F	Fuel temperature coefficient
AM	α_M	Moderator temperature coefficient
AMTRX		Subroutine to form the $\underline{\underline{A}}$ matrix
AMX		Elements of the $\underline{\underline{A}}$ matrix
AX		Coefficients of the EIGEN4 polynomial, i.e., if $\sum_{i=1}^{5} a_n \omega^n$, the a_n.
BE		$\underline{\underline{B}}$ matrix element
BETA	β	Delayed neutron fraction
CK		Function subroutine to calculate the water thermal conductivity as a function of temperature (Equation 2.3.21)
COL	C	Colburn number (Equation 2.3.17)
CP	C_{PM}	Heat capacity of moderator as a function of T_M
CPF	NC_{PF}	Total heat capacity of the fuel
CPFX	C_{PF}	Heat capacity of a single fuel rod
CPG, CPMX	C_{PM}	Total heat capacity of the water
CX, CGUES	k_M	Thermal conductivity of the water
DE	De	Equivalent diameter (Equation 2.3.16)
DELTA	Δ	Convergence criterion for equilibrium calculation

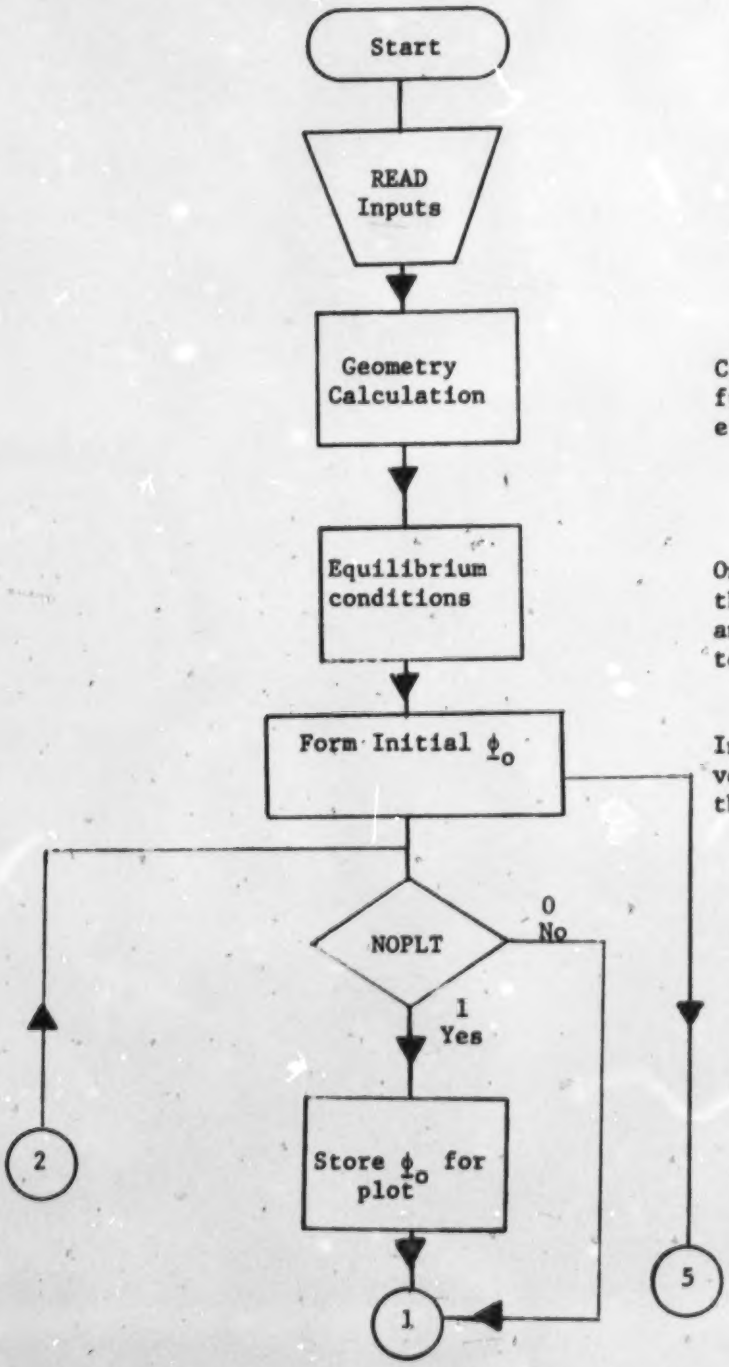
DF	$2R_F$	Fuel diameter
DH	Δt	Time increment (TINCR)
DH1		Lower limit time increment
DH2		Upper limit time increment
DM	t	Time at any instant
DOTMG	$\dot{m}_M$	Mass flow of the coolant
DX, DGUES	ρ_M	Density of the coolant
EIGEN4		Subroutine to change $\lvert\underline{\underline{A}} - \omega\underline{\underline{I}}\rvert = 0$ to polynomial form
EIGENV	ω	Eigenvalues of the $\underline{\underline{A}}$ matrix
FA		Heat transfer area
FCA		Fuel cross section area
FDENS	ρ_F	Fuel density
FH	L	Fuel rod length
FK		Function subroutine used to calculate the fuel conductivity as a function of temperature
FLA		Flow area
FX, FGUES	k_F	Thermal conductivity of fuel
G	γ	Constant in Equation (2.3.12)
GXN		Subroutine to multiply an nxn matrix with a column matrix
H	h_T	Heat transfer coefficient
HH		Elements of the $\underline{\underline{H}}$ matrix (Equation 2.4.19)
HMTRX	$\underline{\underline{H}}$	Subroutine to form the $\underline{\underline{H}}$ matrix
HP		Element of the column matrix $\underline{\underline{H}}\underline{\phi}$
HX, HGUES	h_T	Heat transfer coefficient

MTRX		Order of the nxn matrix
MX	t	Dimensioned time
N		Number of iterations
NA		Number of fuel assemblies
NCHAN		Total number of coolant channels
NFRPA		Number of fuel rods per assembly
NIN		Number of iterations calculated from insertion time and initial time increment
NN		Total number of iterations, calculated from end time and initial time increment
NOPLT		Option for plotting
NR		READ statement unit number
NRO		Option for reactivity insertion
NW		WRITE statement unit number
NZ		Dummy variable used for print out
PAREA		Square pitch area for channel
PC	P_C	Distance between fuel pins
PI	π	3.14159
PKL		$4\pi k_F L_F$
POW	P_o	Input power in MW
PP	P	Power in Btu/hr
PR	Pr	Prandtl number function subroutine
PW	P_w	Wetted perimeter
QQ, QW	Q	Precursor density (Power)
RIN		Reactivity inserted
RB		Element of column matrix $\underline{R}$ $\underline{B}$

RCOS	$\rho = \rho_o \cos a t$	Function subroutine to calculate reactivity for a cosine insertion
RE	Re	Function subroutine to calculate the Reynolds number
RF	R_F	Fuel radius
RHO	$\rho_M(T_M)$	Function subroutine to calculate density of H_2O
RL	$\rho = \rho_o(1+a t)$	Function subroutine to calculate reactivity for a ramp
RMTRX		Subroutine to form the $\underline{\underline{R}}$ matrix (Equation 2.4.20)
RN		Reynolds number
RO	ρ_o	Reactivity inserted at t = 0
RSIN	$\rho = \rho_o \sin a t$	Function subroutine to calculate reactivity for a sine insertion
RP	Pr	Prandtl number
RR	$\rho_T = \rho_{in} + \rho_f$	Total reactivity, inserted plus feedback
RTIME	t_r	Time when inserted reactivity is removed (for ramp reactivity only)
RX		Element of $\underline{\underline{R}}$ matrix (Equation 2.4.20)
RY		Dimensioned reactivity
TCFA		Total cross sectional area of the fuel
TCFLA		Total cross sectional area of coolant
TE		End of calculation time
TFA		Total heat transfer area
TFG		Initial guess for fuel temperature
TFO	T_{FO}	Equilibrium fuel temperature corresponding to P_o
TGUES, TMG		Initial guess for moderator temperature

TMI	T_{MI}	Inlet coolant temperature
TMO	T_{MO}	Equilibrium coolant temperature corresponding to a given P_o
TMOUT		Exit temperature of core water
TPLOT		Subroutine to plot five variables at the same time with respect to the independent variable time
U	μ	Function subroutine to calculate the viscosity of water as a function of temperature
UGUES, UX		Viscosity of water
V	V	Mean velocity of coolant
VF	V_F	Fuel volume
VM	V_M	Coolant volume
WO	ω_o	The largest eigenvalue to the equation $\lvert\underline{\underline{A}} - \omega\underline{\underline{I}}\rvert = 0$
X	λ	Decay constant of the delayed neutron group
XL	Λ	The neutron generation time

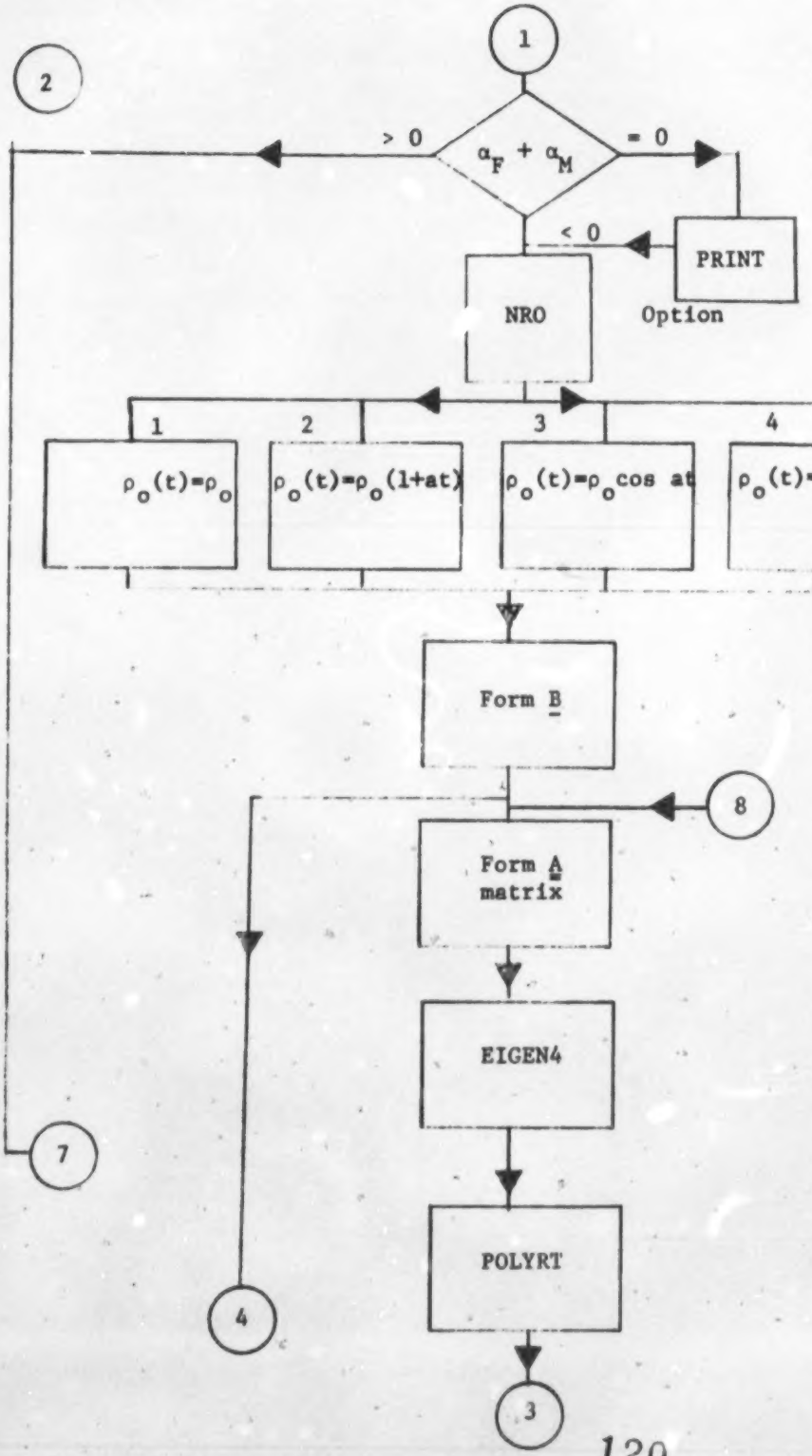
Flow Chart for FUMOTEM



Calculation to obtain fuel area, flow area etc. are necessary

Once P_o is read in, then a consistent fuel and moderator equilibrium temperature is obtained

Initial condition vector is formed from the equilibrium values



If the feedback reactivity coefficients are positive, the program stops. Actually, $\alpha_F + \alpha_M$ must be less than or equal to zero

One of these four reactivity insertions is to be used

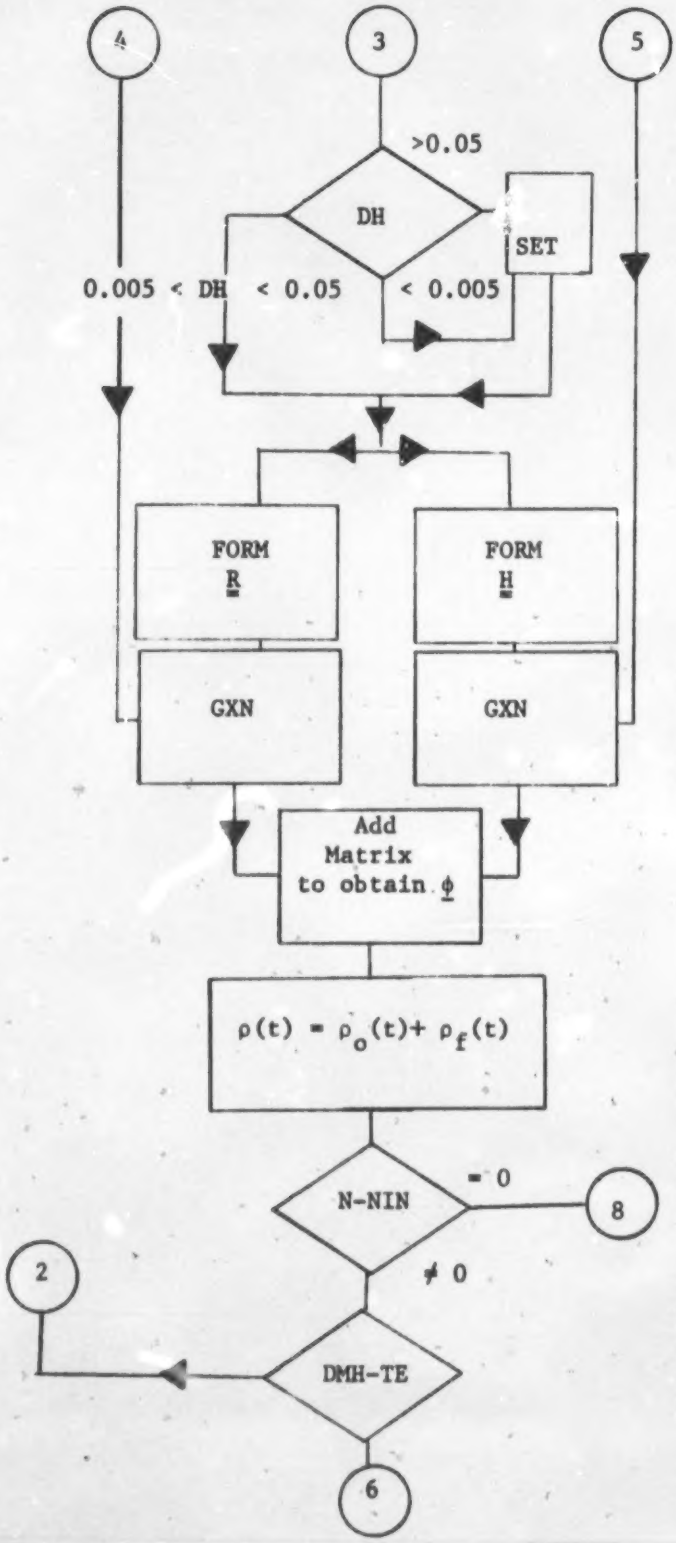
Really, since the $\underline{B}$ vector has 3 zeroes, only the number

$$\frac{2\,\dot{m}_M\,T_{M1}\,(t)}{\rho_M\,V_M}$$

needs to be solved.

This calculates the $|\underline{A} - \omega\underline{I}| = 0$ and puts it into polynomial form

Calculates the largest eigenvalue ω_o



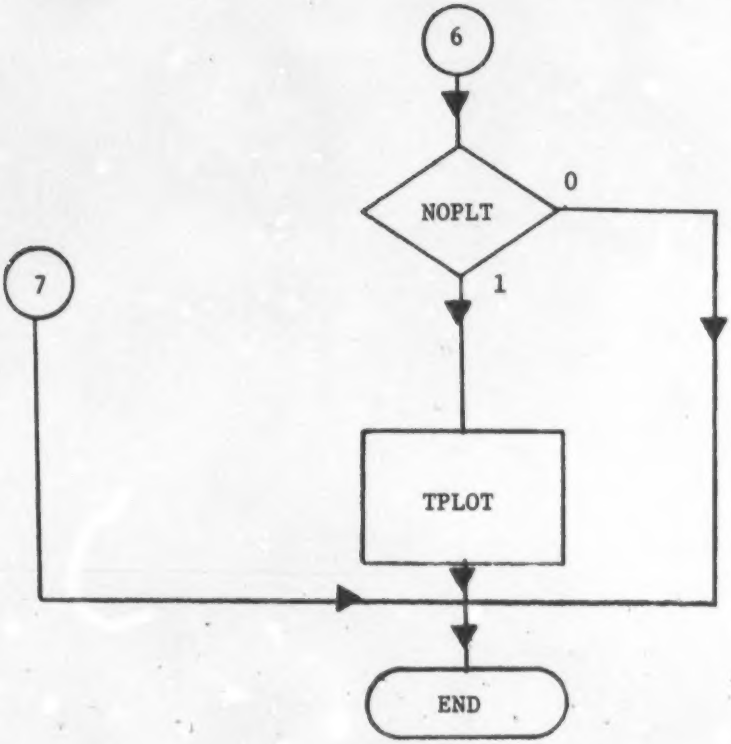
Δt is constrained to remain in the time interval $0.005 < \Delta t < 0.05$

GXN is a subroutine which multiplies matrices to obtain $\underline{\phi}$ (t)

Calculate total reactivity since know inserted and feedback components

If $t \geq t_r$, set $\rho(t) = \rho_f(t)$

If not at end of calculation, go back



This subroutine plots the state vector ϕ as a function of time.

```
//WATFIV      ONEGA,TIME=300,PAGES=50
C     ---------------------------- PWR FEEDBACK ----------------------
C
C
C     CODE NAME     F U M O T E M  .
C
C     OBJECTIVE
C           1.EXAMINE THE TEMPERATURE FEEDBACK MECHANISM OF A PWR AND
C
C           2.SOLVE THE ONE DELAYED NEUTRON MODEL WITH TEMPERATURE
C             FEEDBACK FOR A STEP INSERTION AND A RAMP INSERTION OF
C             REACTIVITY .
C
C     PROGRAM
C           WRITTEN IN SINGLE PRECISION .
C
C
C                 DESCRIPTIONS OF INPUT PARAMETERS .
C
C     FORMAT    PARAMETER          FUNCTION                          UNIT
C     NUMBER
C
C     20        NOPLT        OPTION FOR PLOTTING
C                            1 - PLOT THE RESULT .
C                            0 - NO PLOT .
C
C     30        BETA         AVERAGE FRACTION OF DELAYED NEUTRONS ,
C                            IS 0.00645 , ONLY ONE DELAYED
C                            NEUTRON GROUP IS CONSIDERED .
C               X            DELAYED NEUTRON DECAY CONSTANT .      1/SEC .
C               XL           NEUTRON GENERATION TIME .              SEC
C               RO           INITIAL REACTIVITY INSERTED .
C                            INPUT RO IN DOLLAR UNIT .
C               NRO          OPTION FOR REACTIVITY INSERTION
C                            AS A FUNCTION OF TIME .
C                            1 - CONSTANT REACTIVITY .
C                            2 - LINEAR RAMP INSERTION .
C                            3 - COSINE VARIATION OF REACTIVITY .
C                            4 - SINE VARIATION OF REACTIVITY
C                                WITH TIME
C               RTIME        INSERTION TIME                        SEC
C               A            PERIOD , CONSTANT .                   1/SEC
C
C     40        AM           MODERATOR TEMPERATURE COEFFICIENT      1/F.
C               G            CONSTANT GAMMA IN EQUATION 2.3.13
C                            OF RD-2                               SQRT(F)
C               PTFO         RESONANCE ESCAPE PROBABILITY .
C
C      FUEL ROD PROPERTIES .
C
C     50        RF           RADIUS  .                              FT .
C               PC           DISTANCE BETWEEN RODS .                FT.
C               CPFX         SPECIFIC HEAT .                       BTU/LB-F
C               FDENS        DENSITY .                             LB/FT**3
C               FH           LENGTH .                               FT
C               NA           NUMBER OF FUEL ASSEMBLIES .
C               NFRPA        FUEL RODS PER ASSEMBLY .
C
C     60        V            VELOCITY OF MODERATOR/COOLANT          FT/SEC.
C               TMI          INLET TEMPERATURE OF MODERATOR .       F.
```

```
C                                                                       A  60
C     70        TE          END TIME OF CALCULATION                SEC.  A  61
C               DH          TIME INCREMENT , THE RANGE WILL             A  62
C                           BE BETWEEN 0.005 TO 0.05 SECOND .           A  63
C               DELTA       CONVERGING FACTOR .                         A  64
C                                                                       A  65
C     80        PPW         GIVEN EQUILIBRIUM POWER AT T=0.0       MW   A  66
C               TGUES       COOLANT/MODERATOR GUESSED TEMPERATURE  F.   A  67
C                           CORRESPONDING TO PP .                       A  68
C               TFG         FUEL GUESSED TEMPERATURE CORRESPON  -  F.   A  69
C                           DING TO PP .                                A  70
C                                                                       A  71
C                  MAIN PARAMETERS OTHER THAN INPUT .                   A  72
C                                                                       A  73
C                                                                       A  74
C     POWER DENSITY .                          PHI(1),MPE,MPEL          A  75
C     PRECURSOR DENSITY .                      PHI(2),MQU               A  76
C     FUEL TEMPERATURE .                       PHI(3),MTEEF             A  77
C     MODERATOR/COOLANT TEMPERATURE .          PHI(4),MTEEM             A  78
C     REACTIVITY AT ANY INSTANT .              RR,RY                    A  79
C     TIME ELAPSED .                           DMH,MX                   A  80
C     TOTAL NUMBER OF ITERATIONS .             NN                       A  81
C     COOLANT/MODERATOR EQUILIBRIUM                                     A  82
C                    TEMPERATURE .             TMO                      A  83
C     FUEL EQUILIBRIUM TEMPERATURE .           TFO                      A  84
C                                                                       A  85
C     HEAT TRANSFER COEFFICIENT .              HX,HGUES                 A  86
C     TOTAL MASS FLOW                          DOTMG                    A  87
C     DUMMY INDICATOR FOR PRINT-OUT THE                                 A  88
C       EQUILIBRIUM CONDITION BEFORE AND                                A  89
C       AFTER REACTIVITY INSERTION .           NZ                       A  90
C     FUEL TEMPERATURE COEFFICIENT .           AAF                      A  91
C     NUMBER OF ITERATION AT ANY TIME .        N                        A  92
C                                                                       A  93
C                                                                       A  94
C     ------------------------------------------------------------      A  95
C                                                                       A  96
C                                                                       A  97
      REAL MPEL(500),MPE(500),MQU(500),MTEEF(500),MTEEM(500),MX(500),RY(  A  98
     1500)                                                              A  99
      DIMENSION AMX(4,4),HH(4,4),RX(4,4)                                A 100
      DIMENSION ROOTI(4),EIGNV(4),COF(5),AX(5)                          A 101
      DIMENSION PHI(4),BE(4),RB(4),HP(4)                                A 102
      COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C  A 103
     1PMX,PI,NROD                                                       A 104
      DOUBLE PRECISION AX,COF,EIGNV,ROOTI                               A 105
C                                                                       A 106
C     --------DEFINE REYNOLDS NUMBER , PRANDTL NUMBER , HEAT TRANSFER CO  A 107
C        EFFICIENTS , FUEL COEFFICIENT TEMPERATURE AND REACTIVITY .     A 108
C                                                                       A 109
      RE(RE1,RE2,RE3,RE4)=RE1*RE2*RE3/RE4                               A 110
      PR(PR1,PR2,PR3)=PR1*PR2/PR3                                       A 111
      H(H1,H2,H3,H4,H5)=H1*H2*(H3**.8)*(H4**.3333)/H5                   A 112
      AF(AF1,AF2,AF3)=-AF1*ALOG(1.0/AF3)/SQRT(AF2)                      A 113
      RT(RT1,RT2,RT3,RT4,RT5,RT6,RT7)=RT1+(RT2+RT3)*RT4-RT5*RT2*RT6+RT7 A 114
C                                                                       A 115
C     --------FUEL AND WATER PROPERTY AS TEMPERATURE DEPENDENT .        A 116
C                                                                       A 117
      FK(T)=0.61021E1-0.46365E-2*T+0.13063E-5*T**2                      A 118
      CK(T)=0.11711+0.13910E-2*T-0.18102E-5*T**2                        A 119
```

```
14          RHO(T)=0.57786E2+0.28018E-1*T-0.88346E-4*T**2                    A 120
15          U(T)=0.85067-0.17501E-2*T+0.11420E-5*T**2                        A 121
16          CP(T)=0.47088E1-0.15753E-1*T+0.17233E-4*T**2                     A 122
      C                                                                      A 123
      C                                                                      A 124
      C     --------- MAKE SURE THAT NN EQUAL TO DIMENSION NUMBER OF         A 125
      C               THE PLOTTED VARIABLE .                                 A 126
      C                                                                      A 127
17          NN=1600                                                          A 128
18          NR=5                                                             A 129
19          NW=6                                                             A 130
20          NZ=0                                                             A 131
21          PI=3.14159                                                       A 132
22          N=1                                                              A 133
23          DM=0.0                                                           A 134
24          DH1=0.005                                                        A 135
25          DH2=0.05                                                         A 136
26          DMH=DM*3600.0                                                    A 137
27          R=0.0                                                            A 138
28          RR=0.0                                                           A 139
29          MX(1)=0.0                                                        A 140
      C                                                                      A 141
      C     --------READ INPUT DATA .                                        A 142
      C                                                                      A 143
30          READ (NR,29) NOPLT                                               A 144
31          READ (NR,30) BETA,X,XL,RO,NRO,RTIME,A                            A 145
32          READ (NR,31) AM,G,PTFO                                           A 146
33          READ (NR,32) RF,PC,CPFX,FDENS,FH,NA,NFRPA                        A 147
34          READ (NR,33) V,TMI                                               A 148
      C                                                                      A 149
      C                                                                      A 150
      C     --------THE INITIAL VALUE OF DH WILL CHANGE ACCORDING TO THE LAR - A 151
      C        GEST EIGEN VALUE OF A-MATRIX .                                A 152
      C                                                                      A 153
      C                                                                      A 154
35          READ (NR,34) TE,DH,DELTA                                         A 155
      C                                                                      A 156
      C     --------READ INITIAL POWER DESIRED AND GUESSED FUEL AND COOLANT  A 157
      C        TEMPERATURE .                                                 A 158
      C                                                                      A 159
36          READ (NR,35) PPW,TGUES,TFG                                       A 160
37          PP=3.412D6*PPW                                                   A 161
      C                                                                      A 162
      C     --------PRINT OUT INPUT DATA AND THE INITIAL .                   A 163
      C                                                                      A 164
38          WRITE (NW,36)                                                    A 165
39          WRITE (NW,37)                                                    A 166
40          WRITE (NW,38) BETA,X,XL,RO                                       A 167
41          RO=RO*BETA                                                       A 168
42          WRITE (NW,39) RTIME,A                                            A 169
43          WRITE (NW,40) AM,G,PTFO                                          A 170
44          WRITE (NW,41) RF,PC,CPFX,FDENS,FH                                A 171
45          WRITE (NW,42) NA,NFRPA                                           A 172
46          WRITE (NW,43) V,TMI                                              A 173
47          WRITE (NW,44) NOPLT,NRO                                          A 174
48          WRITE (NW,45) PPW,TFG,TGUES                                      A 175
49          WRITE (NW,46) TE,DH                                              A 176
50          WRITE (NW,47) DELTA                                              A 177
      C                                                                      A 178
      C     --------CHANGE SECOND TO UNIT HOUR .                            A 179
```

```
      C                                                                 A 180
   51         NDH=IFIX(TE/DH+DH/2.0)                                  A 181
   52         IF (NDH.LE.NN) GO TO 1                                  A 182
   53         WRITE (NW,48)                                           A 183
   54         GO TO 28                                                A 184
   55    1    XL=XL/3600.0                                            A 185
   56         X=X*3600.                                               A 186
   57         V=V*3600.                                               A 187
   58         DH1=DH1/3600.0                                          A 188
   59         DH2=DH2/3600.0                                          A 189
   60         DH=DH/3600.                                             A 190
   61         A=A*3600.0                                              A 191
      C                                                               A 192
      C     --------GEOMETRY CALCULATIONS FOR SQUARE PITCH .          A 193
      C                                                               A 194
   62         DF=2.0*RF                                               A 195
   63         FA=PI*DF*FH                                             A 196
   64         FCA=PI*DF**2/4.0                                        A 197
   65         PAREA=PC*PC                                             A 198
   66         FLA=PAREA-FCA                                           A 199
   67         NROD=NA*NFRPA                                           A 200
   68         NCHAN=NROD                                              A 201
   69         TFA=NROD*FA                                             A 202
   70         TCFA=NROD*FCA                                           A 203
   71         TCFLA=NCHAN*FLA                                         A 204
   72         VF=FCA*FH*NROD                                          A 205
   73         VM=TCFLA*FH                                             A 206
   74         PW=PI*DF                                                A 207
   75         DE=4.0*FLA/PW                                           A 208
   76         CPF=CPFX*FDENS*VF                                       A 209
   77         COL=0.042*PC/DF-0.024                                   A 210
      C                                                               A 211
      C     --------ITERATE FUEL AND COOLANT TEMPERATURE UNTIL IT CONVERGES TO  A 212
      C        THE CORRESPONDING POWER AND ITS PRECURSOR .           A 213
      C                                                               A 214
   78    2    CONTINUE                                                A 215
   79         UGUES=U(TGUES)                                          A 216
   80         DGUES=RHO(TGUES)                                        A 217
   81         DOTMG=DGUES*TCFLA*V                                     A 218
   82         CGUES=CK(TGUES)                                         A 219
   83         FGUES=FK(TFG)                                           A 220
   84         PKL=4.0*PI*FGUES*FH                                     A 221
   85         CPG=CP(TGUES)                                           A 222
   86         TMG=TMI+PP/(2.0*CPG*DOTMG)                              A 223
   87         TFG=TMG+PP/(2.0*PKL*NROD)                               A 224
   88         RN=RE(DGUES,V,DE,UGUES)                                 A 225
   89         RP=PR(CPG,UGUES,CGUES)                                  A 226
   90         HGUES=H(COL,CGUES,RN,RP,DE)                             A 227
   91         PGUES=2.0*CPG*DOTMG*ABS(TGUES-TMI)                      A 228
   92         DP=PGUES-PP                                             A 229
   93         DTM=TGUES-TMG                                           A 230
   94         DELPG=ABS(DP)/PP                                        A 231
      C                                                               A 232
      C     --------SET THE GUESSED POWER AND ITERATE UNTIL CONVERGE TO THE  A 233
      C        CORRESPONDING FUEL AND MODERATOR TEMPERATURE ACCURATE TO THE  A 234
      C        VALUE OF DELTA .                                       A 235
      C                                                               A 236
   95         IF (DELPG-DELTA) 6,6,3                                  A 237
   96    3    IF (DTM) 4,6,5                                          A 238
   97    4    TGUES=TGUES+ABS(DTM)/2.0                                A 239
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```
 98          GO TO 2                                                          A 240
 99     5    TGUES=TGUES-ABS(DTM)/2.0                                         A 241
100          GO TO 2                                                          A 242
101     6    CONTINUE                                                         A 243
102          QQ=BETA*PP/(X*XL)                                                A 244
103          QW=QQ/3.412E6                                                    A 245
       C                                                                      A 246
       C     --------INITIALIZE MATRIX PHI .                                  A 247
       C                                                                      A 248
104          PHI(1)=PP                                                        A 249
105          PHI(2)=QQ                                                        A 250
106          PHI(3)=TFG                                                       A 251
107          PHI(4)=TMG                                                       A 252
108          DOTMG=DGUES*TCFLA*V                                              A 253
       C                                                                      A 254
       C     --------SETUP THE FUEL AND COOLANT TEMPERATURE AT EQUILIBRIUM    A 255
       C        STATE .                                                       A 256
       C                                                                      A 257
109          TMO=TMG                                                          A 258
110          TFO=TFG                                                          A 259
111          RS=RR/BETA                                                       A 260
112          WRITE (6,49)                                                     A 261
113          WRITE (NW,50) PPW,QW,TFO,TMO,RS                                  A 262
114          WRITE (NW,51) RN,RP,COL,DOTMG,HGUES                              A 263
115          WRITE (NW,52)                                                    A 264
116          WRITE (NW,53)                                                    A 265
117     7    CONTINUE                                                         A 266
118          MTRX=4                                                           A 267
       C                                                                      A 268
       C     --------STORE THE VALUE OF POWER , PRECURSOR DENSITY , FUEL AND MO A 269
       C        DERATOR TEMPERATURE , AND REACTIVITY FOR PLOTTING .           A 270
       C                                                                      A 271
119          PPW=PHI(1)/3.412E6                                               A 272
120          QW=PHI(2)/3.412E6                                                A 273
121          TFG=PHI(3)                                                       A 274
122          TMG=PHI(4)                                                       A 275
123          IF (NOPLT.EQ.0) GO TO 8                                          A 276
124          MPE(N)=PPW                                                       A 277
125          MPEL(N)=ALOG10(PPW)                                              A 278
126          MQU(N)=QW                                                        A 279
127          MTEEF(N)=TFG                                                     A 280
128          MTEEM(N)=TMG                                                     A 281
129     8    AAF=AF(G,PHI(3),PTFO)                                            A 282
130          IF (AAF.LT.0.0.AND.AM.LT.0.0) GO TO 10                           A 283
131          IF (AAF+AM) 10,9,28                                              A 284
132     9    WRITE (NW,54)                                                    A 285
133     10   UX=U(PHI(4))                                                     A 286
134          DX=RHO(PHI(4))                                                   A 287
135          CX=CK(PHI(4))                                                    A 288
136          CPMX=CP(PHI(4))                                                  A 289
137          FX=FK(PHI(3))                                                    A 290
138          DOTMG=DX*TCFLA*V                                                 A 291
139          HX=H(COL,CX,RE(DX,V,DE,UX),PR(CPMX,UX,CX),DE)                    A 292
140          TMOUT=2.0*PHI(4)-TMI                                             A 293
       C                                                                      A 294
       C     --------COMPUTE THE REACTIVITY INSERTED AS A FUNCTION OF TIME    A 295
       C        AND REACTIVITY .                                              A 296
       C                                                                      A 297
141          GO TO (11,12,13,14), NRO                                         A 298
142     11   RIN=RO                                                           A 299
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```
143             GO TO 15                                                A 300
144     12      RIN=RO*(1+A*DM)                                         A 301
145             GO TO 15                                                A 302
146     13      RIN=RO*COS(A*DM)                                        A 303
147             GO TO 15                                                A 304
148     14      RIN=RO*SIN(A*DM)                                        A 305
149     15      CONTINUE                                                A 306
150             IF (DMH-RTIME) 19,19,16                                 A 307
151     16      IF (NRO.GT.2) GO TO 19                                  A 308
152             IF (NRO-1) 18,17,18                                     A 309
153     17      RIN=0.0                                                 A 310
154             GO TO 19                                                A 311
155     18      RTMH=RTIME/3600.                                        A 312
156             RIN=RO*(1+A*RTMH)                                       A 313
157             GO TO 19                                                A 314
158     19      RY(N)=RR/BETA                                           A 315
        C                                                               A 316
        C       --------FORM B-MATRIX .                                 A 317
        C                                                               A 318
159             BE(1)=0.0                                               A 319
160             BE(2)=0.0                                               A 320
161             BE(3)=0.0                                               A 321
162             BE(4)=+2.0*DOTMG*TMI/(DX*VM)                            A 322
        C                                                               A 323
        C       --------FORM A-MATRIX .                                 A 324
        C                                                               A 325
163             CALL AMTRX (AMX)                                        A 326
        C                                                               A 327
        C       --------CHANGE A-DETERMINANT TO POLYNOMIAL FORM .       A 328
        C                                                               A 329
164             CALL EIGEN4 (AMX,AX)                                    A 330
165             M=5                                                     A 331
166             M1=M                                                    A 332
167             IF (AX(1)) 22,20,22                                     A 333
168     20      DO 21 JK=1,4                                            A 334
169     21      AX(JK)=AX(JK+1)                                         A 335
170             MTRX=3                                                  A 336
171             M1=4                                                    A 337
        C                                                               A 338
        C       --------COMPUTE THE EIGENVALUE OF THE A-MATRIX .        A 339
        C                                                               A 340
172     22      CALL POLRT (AX,COF,MTRX,EIGNV,ROOTI,IER,M1)             A 341
173             MTRX=4                                                  A 342
174             ML=M1-1                                                 A 343
        C                                                               A 344
        C       --------FIND THE LARGEST EIGENVALUE .                   A 345
        C                                                               A 346
175             WO=EIGNV(1)                                             A 347
176             DO 23 IE=1,ML                                           A 348
177             WE=EIGNV(IE)                                            A 349
178     23      WO=AMAX1(WE,WO)                                         A 350
        C                                                               A 351
        C       --------CALCULATE THE TIME INCREMENT , SUCH THAT THE TIME INCRE   A 352
        C          MENT RECIPROCAL TO THE LARGEST EIGEN-VALUE .         A 353
        C          MAKE SURE THAT THE RANGE IS BETWEEN 0.005 TO 0.05 SECOND , IF   A 354
        C          SMALLER THAN 0.005 SECOND CHANGE THE VALUE TO 0.005 AND IF LAR-   A 355
        C          GER THAN 0.05 SET IT TO 0.05 SECOND .                A 356
        C                                                               A 357
179             DH=ABS(1.0/WO)/10.0                                     A 358
180             DDH=DH                                                  A 359
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181           IF (DDH.LT.DH1) DH=DH1                                          A 360
182           IF (DDH.GT.DH2) DH=DH2                                          A 361
      C                                                                       A 362
      C     --------FORM R-MATRIX .                                          A 363
      C                                                                       A 364
183           CALL RMTRX (RX)                                                 A 365
      C                                                                       A 366
      C     --------FORM H-MATRIX .                                          A 367
      C                                                                       A 368
184           CALL HMTRX (HH)                                                 A 369
      C                                                                       A 370
      C     --------MULTIPLY R-MATRIX WITH COLUMN MATRIX-B AND MULTIPLY       A 371
      C        H-MATRIX WITH COLUMN VECTOR PHI .                            A 372
      C                                                                       A 373
185           CALL GXN (RX,BE,MTRX,RB)                                        A 374
186           CALL GXN (HH,PHI,MTRX,HP)                                       A 375
      C                                                                       A 376
      C     --------FORM NEW PHI-MATRIX BY ADDING THE TWO COLUMN MATRIX .     A 377
      C                                                                       A 378
187           DO 24 IP=1,MTRX                                                 A 379
188     24    PHI(IP)=RB(IP)+HP(IP)                                           A 380
189           WRITE (NW,55) N,DMH,RY(N),PPW,QW,TFG,TMG,TMOUT,AAF              A 381
190           IF (DMH-TE) 25,27,27                                            A 382
      C                                                                       A 383
      C     --------IF MODERATOR TEMPERATURE EXCEEDING 700 DEGREE F ,PRINT A  A 384
      C        WARNING AND GET CUT .                                        A 385
      C                                                                       A 386
191     25    IF (PHI(4).GE.1000.0) GO TO 26                                  A 387
192           RR=AM*(PHI(4)-TMO)+AAF*(PHI(3)-TFO)+RIN                         A 388
193           R=RR                                                            A 389
194           DM=DM+DH                                                        A 390
195           DMH=DM*3600.0                                                   A 391
196           IF (DMH.GE.TE) GO TO 27                                         A 392
197           N=N+1                                                           A 393
198           MX(N)=DMH                                                       A 394
199           GO TO 7                                                         A 395
200     26    WRITE (NW,56)                                                   A 396
201     27    NN=N                                                            A 397
202           IF (NOPLT.EQ.0) GO TO 28                                        A 398
      C                                                                       A 399
      C     --------PLOT THE RESULT SIMULTANEOUSLY IN ONE GRAPH .             A 400
203           CALL TPLOT (MX,MPE,MPEL,MQU,MTEEM,MTEEF,RY,NN)                  A 401
204     28    STOP                                                            A 402
      C                                                                       A 403
205     29    FORMAT (I2)                                                     A 404
206     30    FORMAT (2F10.5,2F10.7,I10,2F10.5)                               A 405
207     31    FORMAT (3F10.6)                                                 A 406
208     32    FORMAT (3F10.5,2F10.2,2I10)                                     A 407
209     33    FORMAT (2F10.2)                                                 A 408
210     34    FORMAT (3F10.4)                                                 A 409
211     35    FORMAT (3F10.2)                                                 A 410
212     36    FORMAT (1H1)                                                    A 411
213     37    FORMAT (1X,23H**********************,/,1X,19HMODULE 2 , FEEDBACK,  A 412
             1/,1X,23H**********************,/////,1X,10HINPUT DATA,/,1X,14H***  A 413
             2**********,///)                                                 A 414
214     38    FORMAT (5X,4HBETA,5X,6HLAMBDA,5X,17HNEUTRON GEN. TIME,5X,18HINITIA  A 415
             1L REACTIVITY,/,12X,11H( SEC**-1 ),6X,7H( SEC ),18X,5H( $ ),/,3X,F8  A 416
             2.6,3X,F7.5,7X,E9.3,17X,F5.2,/)                                  A 417
215     39    FORMAT (5X,14HINSERTION TIME,5X,15HCONSTANT PERIOD,/,9X,5H(SEC),14  A 418
             1X,7H(1/SEC),/,8X,F7.3,13X,F7.3,/)                               A 419
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      40    FORMAT (5X,14HCOOLANT COEFF.,5X,10HCONSTANT B,5X,20HRESONANCE ESC.   A 420
     1 PROB.,/,6X,12H(          ),7X,8H(       ),7X,18H(                 )    A 421
     2,/,6X,E12.5,7X,E8.2,13X,F5.3,/)                                          A 422
      41    FORMAT (5X,11HFUEL RADIUS,5X,5HPITCH,5X,8HFUEL CP.,5X,12HFUEL DENS  A 423
     1ITY,5X,11HFUEL HEIGHT,5X,18H                    ,5X,12H          ,      A 424
     2/,7X,6H( FT ),8X,6H( FT ),2X,12H( BTU/LB-F ),3X,12H( LB/FT**3 ),7X       A 425
     3,6H( FT ),9X,16H                ,7X,10H          ,/,7X,F7.5,7X,F7.      A 426
     45,4X,F6.4,8X,F6.2,10X,F6.2,/)                                            A 427
      42    FORMAT (5X,18HNUMBER OF ASSEMBLY,5X,26HNUMBER OF ROD PER ASSEMBLY,   A 428
     1/,5X,18H(                ),5X,26H(                          ),/,12X,     A 429
     2I4,23X,I4,/)                                                             A 430
      43    FORMAT (5X,16HCOOLANT VELOCITY,5X,20HINLET COOLANT TEMPT.,/,8X,10H   A 431
     1( FT/SEC ),15X,5H( F ),/,11X,F4.1,18X,F5.1,/)                            A 432
      44    FORMAT (9X,6HOPTION,4X,4HPLOT,9X,I1,/,19X,11HTYPE INSER.,2X,I1,/)    A 433
      45    FORMAT (5X,13HINITIAL POWER,9X,4H(MW),1X,E11.4,/,5X,19HGUESSED FUE   A 434
     1L TEMPT.,4X,3H(F),5X,F6.1,/,5X,22HGUESSED COOLANT TEMPT.,1X,3H(F),       A 435
     26X,F5.1,///,2X,17HEND OF INPUT DATA,/,2X,18(1H*),///)                    A 436
      46    FORMAT (//,5X,12H   END TIME ,3X,14HTIME INCREMENT,/,8X,F6.3,10X,F   A 437
     16.4,//)                                                                  A 438
      47    FORMAT (5X,22H CONVERGENCE FACTOR = ,F9.5,//)                       A 439
      48    FORMAT (//,5X,118H***  CHECK DIMENSION , IFIX(TE/DH) HAS TO BE SMA   A 440
     1LLER OR EQUAL TO THE DIMENSION OF MPEL,MPE,MTEEF,MTEEM,MY,RY AND N       A 441
     2QU .,///)                                                                A 442
      49    FORMAT (///,5X,27HEQUILIBRIUM STATE INITIALLY,/,4X,28(1H*),///)     A 443
      50    FORMAT (5X,5HPOWER,5X,9HPRECURSOR,5X,11HFUEL TEMPT.,5X,14HCOOLANT    A 444
     1TEMPT.,5X,10HREACTIVITY,5X,11H           ,/,3X,E10.4,2X,E10.4,6X,F       A 445
     27.2,11X,F6.2,11X,F6.3,////)                                              A 446
      51    FORMAT (85(1H*),////,5X,10HREYNOLDS #,3X,9HPRANDTL #,3X,9HCOLBURN    A 447
     1#,3X,9HMASS FLOW,3X,19HHEAT TRANFER COEFF.,/,42X,9H( LB/HR ),4X,18       A 448
     2H( BTU/HR-F-FT**2 ),/,6X,F8.1,6X,F4.1,5X,F7.3,5X,E11.4,8X,F7.1,///       A 449
     3)                                                                        A 450
      52    FORMAT (5X,2HNO,10X,4HTIME,6X,10HREACTIVITY,7X,5HPOWER,11X,9HPRECU   A 451
     1RSOR,/,4X,4H(  ),8X,5H(SEC),9X,3H($),11X,4H(MW),14X,4H(MW))              A 452
      53    FORMAT (7X,9HFUEL TEMP,5X,9HMOD. TEMP,3X,9HEXIT TEMP,8X,11HFUEL CO   A 453
     1EFF.,/,10X,3H(F),11X,3H(F),9X,3H(F),14X,5H(1/F),//)                      A 454
      54    FORMAT (//,8X,19H... NO FEEDBACK ...,//)                            A 455
      55    FORMAT (3X,I4,8X,F7.3,7X,F5.2,7X,E11.4,7X,E11.4,/,8X,F7.2,7X,F7.2,   A 456
     15X,F7.2,9X,E11.4)                                                        A 457
      56    FORMAT (//,5X,39H C R I T I C A L  T E M P E R A T U R E,//)        A 458
            END                                                               A 459


            SUBROUTINE RMTRX (R)                                              B   1
      C                                                                       B   2
      C                                                                       B   3
      C     --------FORM R-MATRIX , DIAGONAL MATRIX IN EQUATION 2.4.20         B   4
      C        SUPPORTING ROUTINE  NONE                                       B   5
      C                                                                       B   6
      C                                                                       B   7
            DIMENSION R(4,4),A(4,4)                                           B   8
            COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C  B   9
           1PMX,PI,NROD                                                       B  10
            CALL AMTRX (A)                                                    B  11
            DO 3 I=1,4                                                        B  12
            DO 3 II=1,4                                                       B  13
            IF (I-II) 1,2,1                                                   B  14
      1     R(I,II)=0.0                                                       B  15
            GO TO 3                                                           B  16
      2     R(I,II)=(EXP(A(I,II)*DH)-1.0)/A(I,II)                             B  17
      3     CONTINUE                                                          B  18
```

```
      RETURN                                                            B  19
      END                                                               B  20


      SUBROUTINE HMTRX (H)                                              C   1
C                                                                       C   2
C     --------FORM H-MATRIX AS IN THE EQUATION 2.4.19 .                 C   3
C        SUPPORTING ROUTINE  NONE                                       C   4
C                                                                       C   5
C                                                                       C   6
      DIMENSION H(4,4),A(4,4)                                           C   7
      COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C C   8
     1PMX,PI,NROD                                                       C   9
      CALL AMTRX (A)                                                    C  10
      H(1,1)=EXP(A(1,1)*DH)                                             C  11
      H(1,2)=A(1,2)*(EXP(WO*DH)-EXP(A(1,1)*DH))/(WO-A(1,1))             C  12
      H(1,3)=0.0                                                        C  13
      H(1,4)=0.0                                                        C  14
      H(2,1)=A(2,1)*(EXP(WO*DH)-EXP(A(2,2)*DH))/(WO-A(2,2))             C  15
      H(2,2)=EXP(A(2,2)*DH)                                             C  16
      H(2,3)=0.0                                                        C  17
      H(2,4)=0.0                                                        C  18
      H(3,1)=A(3,1)*(EXP(WO*DH)-EXP(A(3,3)*DH))/(WO-A(3,3))             C  19
      H(3,2)=0.0                                                        C  20
      H(3,3)=EXP(A(3,3)*DH)                                             C  21
      H(3,4)=A(3,4)*(EXP(WO*DH)-EXP(A(3,3)*DH))/(WO-A(3,3))             C  22
      H(4,1)=A(4,1)*(EXP(WO*DH)-EXP(A(4,4)*DH))/(WO-A(4,4))             C  23
      H(4,2)=0.0                                                        C  24
      H(4,3)=0.0                                                        C  25
      H(4,4)=EXP(A(4,4)*DH)                                             C  26
      RETURN                                                            C  27
      END                                                               C  28


      SUBROUTINE POLRT (XCOF,COF,M,ROOTR,ROOTI,IER,M1)                  D   1
C                                                                       D   2
C                                                                       D   3
C        SUBROUTINE POLRT                                               D   4
C                                                                       D   5
C        PURPOSE                                                        D   6
C           COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL    D   7
C                                                                       D   8
C        USAGE                                                          D   9
C           CALL POLRT(XCOF,COF,M,ROOTR,ROOTI,IER,M1)                   D  10
C                                                                       D  11
C        DESCRIPTION OF PARAMETERS                                      D  12
C           XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL          D  13
C                 ORDERED FROM SMALLEST TO LARGEST POWER                D  14
C           COF  -WORKING VECTOR OF LENGTH M+1                          D  15
C           M    -ORDER OF POLYNOMIAL                                   D  16
C           ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS    D  17
C                 OF THE POLYNOMIAL                                     D  18
C           ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE           D  19
C                 CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL       D  20
C           IER  -ERROR CODE WHERE                                      D  21
C                 IER=0  NO ERROR                                       D  22
C                 IER=1  M LESS THAN ONE                                D  23
C                 IER=2  M GREATER THAN 36                              D  24
C                 IER=3  UNABLE TO DETERMINE ROOT WITH 500 INTERATIONS  D  25
C                        ON 5 STARTING VALUES                           D  26
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C                 IER=4  HIGH ORDER COEFFICIENT IS ZERO                    D  27
C           M1   -NUMBER OF COEFFICIENT , M+1                              D  28
C                (ADDED ARGUMENT FROM THE ORIGINAL TO GET MORE FLEXIBLE    D  29
C                                                      DIMENSION )         D  30
C                                                                          D  31
C        REMARKS                                                           D  32
C           LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.                      D  33
C           FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER               D  34
C           POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE RESULTS.   D  35
C                                                                          D  36
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                     D  37
C           NONE                                                           D  38
C                                                                          D  39
C        METHOD                                                            D  40
C           NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS      D  41
C           ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL       D  42
C           RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED        D  43
C           ERRORS IN THE REDUCED POLYNOMIAL.                              D  44
C                                                                          D  45
C                                                                          D  46
      DIMENSION XCOF(M1),COF(M1),ROOTR(M),ROOTI(M)                         D  47
      DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,    D  48
     1 DX,DY,TEMP,ALPHA,DABS                                               D  49
C                                                                          D  50
C                                                                          D  51
C       --------IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED,  D  52
C          C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION       D  53
C          STATEMENT WHICH FOLLOWS.                                        D  54
C                                                                          D  55
      DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI                                D  56
C                                                                          D  57
C       --------THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENT  D  58
C          APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS       D  59
C          ROUTINE.                                                        D  60
C          THE DOUBLE PRECISION VERSION MAY BE MODIFIED BY CHANGING THE    D  61
C          CONSTANT IN STATEMENT 78 TO 1.0D-12 AND IN STATEMENT 122 TO     D  62
C          1.0D-10.  THIS WILL PROVIDE HIGHER PRECISION RESULTS AT THE     D  63
C          COST OF EXECUTION TIME                                          D  64
C                                                                          D  65
C                                                                          D  66
      IFIT=0                                                               D  67
      N=M                                                                  D  68
      IER=0                                                                D  69
      IF (XCOF(N+1)) 1,4,1                                                 D  70
    1 IF (N) 2,2,6                                                         D  71
C                                                                          D  72
C       --------SET ERROR CODE TO 1                                        D  73
C                                                                          D  74
    2 IER=1                                                                D  75
    3 RETURN                                                               D  76
C                                                                          D  77
C       --------SET ERROR CODE TO 4                                        D  78
C                                                                          D  79
    4 IER=4                                                                D  80
      GO TO 3                                                              D  81
C                                                                          D  82
C       --------SET ERROR CODE TO 2                                        D  83
C                                                                          D  84
    5 IER=2                                                                D  85
      GO TO 3                                                              D  86
```

```
284   6     IF (N-36) 7,7,5                                          D  87
285   7     NX=N                                                     D  88
286         NXX=N+1                                                  D  89
287         N2=1                                                     D  90
288         KJ1=N+1                                                  D  91
289         DO 8 L=1,KJ1                                             D  92
290         MT=KJ1-L+1                                               D  93
291   8     COF(MT)=XCOF(L)                                          D  94
      C                                                              D  95
      C     --------SET INITIAL VALUES                               D  96
      C                                                              D  97
292   9     XO=.00500101                                             D  98
293         YO=0.01000101                                            D  99
      C                                                              D 100
      C     --------ZERO INITIAL VALUE COUNTER                       D 101
      C                                                              D 102
294         IN=0                                                     D 103
295   10    X=XO                                                     D 104
      C                                                              D 105
      C     --------INCREMENT INITIAL VALUES AND COUNTER            D 106
      C                                                              D 107
296         XO=-10.0*YO                                              D 108
297         YO=-10.0*X                                               D 109
      C                                                              D 110
      C     --------SET X AND Y TO CURRENT VALUE                    D 111
      C                                                              D 112
298         X=XO                                                     D 113
299         Y=YO                                                     D 114
300         IN=IN+1                                                  D 115
301         GO TO 12                                                 D 116
302   11    IFIT=1                                                   D 117
303         XPR=X                                                    D 118
304         YPR=Y                                                    D 119
      C                                                              D 120
      C     --------EVALUATE POLYNOMIAL AND DERIVATIVES              D 121
      C                                                              D 122
305   12    ICT=0                                                    D 123
306   13    UX=0.0                                                   D 124
307         UY=0.0                                                   D 125
308         V=0.0                                                    D 126
309         YT=0.0                                                   D 127
310         XT=1.0                                                   D 128
311         U=COF(N+1)                                               D 129
312         IF (U) 14,27,14                                          D 130
313   14    DO 15 I=1,N                                              D 131
314         L=N-I+1                                                  D 132
315         TEMP=COF(L)                                              D 133
316         XT2=X*XT-Y*YT                                            D 134
317         YT2=X*YT+Y*XT                                            D 135
318         U=U+TEMP*XT2                                             D 136
319         V=V+TEMP*YT2                                             D 137
320         FI=I                                                     D 138
321         UX=UX+FI*XT*TEMP                                         D 139
322         UY=UY-FI*YT*TEMP                                         D 140
323         XT=XT2                                                   D 141
324   15    YT=YT2                                                   D 142
325         SUMSQ=UX*UX+UY*UY                                        D 143
326         IF (SUMSQ) 16,23,16                                      D 144
327   16    DX=(V*UY-U*UX)/SUMSQ                                     D 145
328         X=X+DX                                                   D 146
```

```
      DY=-(U*UY+V*UX)/SUMSQ                                              D 147
      Y=Y+DY                                                             D 148
      IF (DABS(DY)+DABS(DX)-1.0D-5) 21,17,17                             D 149
C                                                                        D 150
C     --------STEP ITERATION COUNTER                                     D 151
C                                                                        D 152
17    ICT=ICT+1                                                          D 153
      IF (ICT-500) 13,18,18                                              D 154
18    IF (IFIT) 21,19,21                                                 D 155
19    IF (IN-5) 10,20,20                                                 D 156
C                                                                        D 157
C     --------SET ERROR CODE TO 3                                        D 158
C                                                                        D 159
20    IER=3                                                              D 160
      GO TO 3                                                            D 161
21    DO 22 L=1,NXX                                                      D 162
      MT=KJ1-L+1                                                         D 163
      TEMP=XCOF(MT)                                                      D 164
      XCOF(MT)=COF(L)                                                    D 165
22    COF(L)=TEMP                                                        D 166
      ITEMP=N                                                            D 167
      N=NX                                                               D 168
      NX=ITEMP                                                           D 169
      IF (IFIT) 25,11,25                                                 D 170
23    IF (IFIT) 24,10,24                                                 D 171
24    X=XPR                                                              D 172
      Y=YPR                                                              D 173
25    IFIT=0                                                             D 174
      IF (DABS(Y)-1.0D-4*DABS(X)) 28,26,26                               D 175
26    ALPHA=X+X                                                          D 176
      SUMSQ=X*X+Y*Y                                                      D 177
      N=N-2                                                              D 178
      GO TO 29                                                           D 179
27    X=0.0                                                              D 180
      NX=NX-1                                                            D 181
      NXX=NXX-1                                                          D 182
28    Y=0.0                                                              D 183
      SUMSQ=0.0                                                          D 184
      ALPHA=X                                                            D 185
      N=N-1                                                              D 186
29    COF(2)=COF(2)+ALPHA*COF(1)                                         D 187
      IF (N.EQ.0) GO TO 31                                               D 188
      DO 30 L=2,N                                                        D 189
30    COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)                      D 190
31    ROOTI(N2)=Y                                                        D 191
      ROOTR(N2)=X                                                        D 192
      N2=N2+1                                                            D 193
      IF (SUMSQ) 32,33,32                                                D 194
32    Y=-Y                                                               D 195
      SUMSQ=0.0                                                          D 196
      GO TO 31                                                           D 197
33    IF (N) 3,3,9                                                       D 198
      END                                                                D 199


      SUBROUTINE TPLOT (M1,M0,M2,M5,M6,M8,M9,JX)                         E   1
C                                                                        E   2
C                                                                        E   3
C     --------TPLOT IS PLOTTING SEVERAL VARIABLES IN ONE GRAPH . THE X-A E   4
C        DUES NOT REPRESENT ANY VARIABLE , IT IS INTEGER SEQUENCES .     E   5
```

```
      C        FOR NEGATIVE VALUES , THE ZERO LINE IS IN THE 0.5 LINE .          E   6
      C                                                                          E   7
      C        SUPPORTING ROUTINE  NONE                                          E   8
      C                                                                          E   9
  377         IMPLICIT REAL*4(A-H,M-Z)                                         E  10
  378         DIMENSION M8(JX),M9(JX),M0(JX),M1(JX),M2(JX),M5(JX),M6(JX)       E  11
  379         DIMENSION LINE(61),INUM(9)                                       E  12
  380         INTEGER PL,MI,S5,BL,SL,S9,S0,S6,S1,S2                            E  13
  381         READ (5,8) PL,MI,S5,BL,SL,S9,S0,S1,S2,S6                         E  14
  382         MXY=0.0                                                          E  15
  383         MIN0=0.0                                                         E  16
  384         MIN5=0.0                                                         E  17
  385         MIN6=0.0                                                         E  18
  386         MIN2=0.0                                                         E  19
  387         MIN8=0.0                                                         E  20
  388         MIN9=0.0                                                         E  21
  389         PHI5=0.0                                                         E  22
  390         PHI6=0.0                                                         E  23
  391         PHI0=0.0                                                         E  24
  392         PHI2=0.0                                                         E  25
  393         PHI8=0.0                                                         E  26
  394         PHI9=0.0                                                         E  27
  395         DO 1 I=1,JX                                                      E  28
  396         IF (MIN0.GT.M0(I)) MIN0=M0(I)                                    E  29
  397         IF (MIN5.GT.M5(I)) MIN5=M5(I)                                    E  30
  398         IF (MIN6.GT.M6(I)) MIN6=M6(I)                                    E  31
  399         IF (MIN2.GT.M2(I)) MIN2=M2(I)                                    E  32
  400         IF (MIN8.GT.M8(I)) MIN8=M8(I)                                    E  33
  401         IF (MIN9.GT.M9(I)) MIN9=M9(I)                                    E  34
  402         IF (ABS(M0(I)).GT.PHI0) PHI0=ABS(M0(I))                          E  35
  403         IF (ABS(M2(I)).GT.PHI2) PHI2=ABS(M2(I))                          E  36
  404         IF (ABS(M8(I)).GT.PHI8) PHI8=ABS(M8(I))                          E  37
  405         IF (ABS(M9(I)).GT.PHI9) PHI9=ABS(M9(I))                          E  38
  406         IF (ABS(M5(I)).GT.PHI5) PHI5=ABS(M5(I))                          E  39
  407         IF (ABS(M6(I)).GT.PHI6) PHI6=ABS(M6(I))                          E  40
  408    1    CONTINUE                                                         E  41
  409         JJ=JX                                                            E  42
  410         JJ0=JJ*6+1                                                       E  43
  411         JJ1=JJ+1                                                         E  44
  412         WRITE (6,9)                                                      E  45
  413         WRITE (6,10)                                                     E  46
  414         PHI0=PHI0+ABS(MIN0)                                              E  47
  415         PHI5=PHI5+ABS(MIN5)                                              E  48
  416         PHI6=PHI6+ABS(MIN6)                                              E  49
  417         PHI2=PHI2+ABS(MIN2)                                              E  50
  418         PHI8=PHI8+ABS(MIN8)                                              E  51
  419         PHI9=PHI9+ABS(MIN9)                                              E  52
  420         DO 2 I=1,JJ                                                      E  53
  421         IF (MIN0.LT.0.0) M0(I)=M0(I)+ABS(MIN0)                           E  54
  422         IF (MIN5.LT.0.0) M5(I)=M5(I)+ABS(MIN5)                           E  55
  423         IF (MIN6.LT.0.0) M6(I)=M6(I)+ABS(MIN6)                           E  56
  424         IF (MIN2.LT.0.0) M2(I)=M2(I)+ABS(MIN2)                           E  57
  425         IF (MIN8.LT.0.0) M8(I)=M8(I)+ABS(MIN8)                           E  58
  426         IF (MIN9.LT.0.0) M9(I)=M9(I)+ABS(MIN9)                           E  59
  427         M0(I)=M0(I)/PHI0                                                 E  60
  428         M5(I)=M5(I)/PHI5                                                 E  61
  429         M6(I)=M6(I)/PHI6                                                 E  62
  430         M2(I)=M2(I)/PHI2                                                 E  63
  431         M8(I)=M8(I)/PHI8                                                 E  64
  432    2    M9(I)=M9(I)/PHI9                                                 E  65
```

```
433           DO 3 I=1,9                                                E  66
434     3     INUM(I)=I                                                 E  67
435           WRITE (6,11) (INUM(I),I=1,9)                              E  68
436           DO 7 I=1,JJ1                                              E  69
437           IF (I.EQ.1) GO TO 5                                       E  70
438           MXY=M1(I-1)                                               E  71
439           IP8=M8(I-1)*60+1.0                                        E  72
440           IP5=M5(I-1)*60+1.0                                        E  73
441           IP9=M9(I-1)*60+1.0                                        E  74
442           IP6=M6(I-1)*60+1.0                                        E  75
443           IP0=M0(I-1)*60+1.0                                        E  76
444           IP2=M2(I-1)*60+1.0                                        E  77
445           DO 4 I1=1,56,5                                            E  78
446           LINE(I1)=BL                                               E  79
447           DO 4 I2=1,4                                               E  80
448           I3=I1+I2                                                  E  81
449           IF (I1.EQ.IP0) LINE(I1)=S0                                E  82
450           IF (I3.EQ.IP0) LINE(I3)=S0                                E  83
451           IF (I1.EQ.IP2) LINE(I1)=S2                                E  84
452           IF (I3.EQ.IP2) LINE(I3)=S2                                E  85
453           IF (I1.EQ.IP5) LINE(I1)=S5                                E  86
454           IF (I3.EQ.IP5) LINE(I3)=S5                                E  87
455           IF (I1.EQ.IP6) LINE(I1)=S6                                E  88
456           IF (I3.EQ.IP6) LINE(I3)=S6                                E  89
457           IF (I1.EQ.IP8) LINE(I1)=SL                                E  90
458           IF (I3.EQ.IP8) LINE(I3)=SL                                E  91
459           IF (I1.EQ.IP9) LINE(I1)=S9                                E  92
460           IF (I3.EQ.IP9) LINE(I3)=S9                                E  93
461     4     CONTINUE                                                  E  94
462           LINE(61)=PL                                               E  95
463           IIL=I-1                                                   E  96
464           IF (1.EQ.IP0) LINE(1)=S0                                  E  97
465           IF (1.EQ.IP2) LINE(1)=S2                                  E  98
466           IF (1.EQ.IP5) LINE(1)=S5                                  E  99
467           IF (1.EQ.IP6) LINE(1)=S6                                  E 100
468           IF (1.EQ.IP8) LINE(1)=SL                                  E 101
469           IF (1.EQ.IP9) LINE(1)=S9                                  E 102
470           IF (IP0.EQ.61) LINE(61)=S0                                E 103
471           IF (IP5.EQ.61) LINE(61)=S5                                E 104
472           IF (IP2.EQ.61) LINE(61)=S2                                E 105
473           IF (IP6.EQ.61) LINE(61)=S6                                E 106
474           IF (IP8.EQ.61) LINE(61)=SL                                E 107
475           IF (IP9.EQ.61) LINE(61)=S9                                E 108
476           IF (IP0.NE.1.OR.IP2.NE.1.OR.IP5.NE.1.OR.IP6.NE.1.OR.IP8.NE.1.OR.IP  E 109
             19.NE.1) LINE(1)=PL                                        E 110
477           WRITE (6,12) MXY,(LINE(KK),KK=1,61)                       E 111
478           IF (I.EQ.JJ1) GO TO 7                                     E 112
479     5     CONTINUE                                                  E 113
480           DO 6 I1=1,56,5                                            E 114
481           DO 6 I2=1,4                                               E 115
482           I3=I1+I2                                                  E 116
483           LINE(I3)=BL                                               E 117
484     6     CONTINUE                                                  E 118
485     7     CONTINUE                                                  E 119
486           WRITE (6,13) (INUM(I),I=1,9)                              E 120
487           WRITE (6,15) PL,MI,S5,BL,SL,S9,S0,S1,S2,S6                E 121
488           WRITE (6,16)                                              E 122
489           WRITE (6,14)                                              E 123
490           STOP                                                      E 124
      C                                                                 E 125
```

```
      8     FORMAT (11A1)                                                    E 126
      9     FORMAT (1H1)                                                     E 127
     10     FORMAT (35X,16HRELATIVE DENSITY)                                 E 128
     11     FORMAT (17X,9(2X,2HO.,I1,1H )/,14X,10(6H+-----),1H+)             E 129
     12     FORMAT (1X,4HTIME,1X,F7.3,1X,61A1)                               E 130
     13     FORMAT (14X,10(6H+-----),1H+,/,16X,9(3X,2HO.,I1))                E 131
     14     FORMAT (1H1)                                                     E 132
     15     FORMAT (15X,17HINPUT CHARACTER  ,11A1,/)                         E 133
     16     FORMAT (26H                          ,/,15X,10H P - POWER,/,15X,18  E 134
    1H L - LOG. OF POWER,/,15X,22H Q - PRECURSOR DENSITY,/,15X,21H F -       E 135
    2FUEL TEMPERATURE,/,15X,26H M - MODERATOR TEMPERATURE,/,15X,15H R -      E 136
    3 REACTIVITY,/,15X,42HIF VARIABLE PLOTTED HAS NEGATIVE VALUE THE,/,      E 137
    415X,41HAXIS FOR THIS VARIABLE IS SHIFTED TO THE ,/,15X,16HCENTER O      E 138
    5F Y-AXIS,///)                                                           E 139
            END                                                              E 140


            SUBROUTINE GXN (G,XN,N,GX)                                       F   1
      C                                                                      F   2
      C                                                                      F   3
      C     --------------MULTIPLY  THE G - MATRIX WITH THE INITIAL VECTOR CO F   4
      C               LUMN TO GET THE NEXT ITERATION .                       F   5
      C                                                                      F   6
      C        SUPPORTING ROUTINE  NONE                                      F   7
      C                                                                      F   8
      C                                                                      F   9
            DIMENSION G(N,N),XN(N),GX(N)                                     F  10
            DO 2 I=1,N                                                       F  11
            GG=0.0                                                           F  12
            DO 1 J=1,N                                                       F  13
      1     GG=GG+G(I,J)*XN(J)                                               F  14
      2     GX(I)=GG                                                         F  15
            RETURN                                                           F  16
            END                                                              F  17


            SUBROUTINE AMTRX (A)                                             G   1
      C                                                                      G   2
      C                                                                      G   3
      C     --------FORM A-MATRIX IN EQUATION 2.4.12 .                       G   4
      C        SUPPORTING ROUTINE  NONE                                      G   5
      C                                                                      G   6
      C                                                                      G   7
            DIMENSION A(4,4)                                                 G   8
            COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C  G   9
           1PMX,PI,NROD                                                      G  10
            A(1,1)=(RR-BETA)/XL                                              G  11
            A(1,2)=X                                                         G  12
            A(1,3)=0.0                                                       G  13
            A(1,4)=0.0                                                       G  14
            A(2,1)=BETA/XL                                                   G  15
            A(2,2)=-X                                                        G  16
            A(2,3)=0.0                                                       G  17
            A(2,4)=0.0                                                       G  18
            A(3,1)=1.0/(2.0*CPF)                                             G  19
            A(3,2)=0.0                                                       G  20
            A(3,3)=-4.0*PI*FX*FH*NROD/CPF                                    G  21
            A(3,4)=-A(3,3)                                                   G  22
            A(4,1)=1.0/(DX*CPMX*VM)                                          G  23
            A(4,2)=0.0                                                       G  24
```

```
      A(4,3)=0.0                                                        G  25
      A(4,4)=-2.0*DOTMG/(DX*VM)                                         G  26
      RETURN                                                            G  27
      END                                                               G  28

      SUBROUTINE EIGEN4 (A,B)                                           H   1
C                                                                       H   2
C                                                                       H   3
C     --------EIGEN4 IS SUBSTITUTING DETERMINANT A TO POLYNOMIAL FORM .  H   4
C        SUPPORTING ROUTINE  NONE                                       H   5
C                                                                       H   6
C                                                                       H   7
      DIMENSION A(4,4),B(5)                                             H   8
      DOUBLE PRECISION B                                                H   9
      B(1)=A(1,1)*A(2,2)*A(3,3)*A(4,4)-A(1,2)*A(2,1)*A(3,3)*A(4,4)      H  10
      B(2)=-(A(1,1)*A(2,2)*A(3,3)+A(1,1)*A(2,2)*A(4,4)+A(1,1)*A(3,3)*A(4 H  11
     1,4)+A(2,2)*A(3,3)*A(4,4)-A(1,2)*A(2,1)*A(3,3)-A(1,2)*A(2,1)*A(4,4) H  12
     2)                                                                 H  13
      B(3)=A(3,3)*A(4,4)+A(1,1)*A(2,2)+A(1,1)*A(3,3)+A(1,1)*A(4,4)+A(2,2 H  14
     1)*A(3,3)+A(2,2)*A(4,4)-A(1,2)*A(2,1)                              H  15
      B(4)=-(A(1,1)+A(2,2)+A(3,3)+A(4,4))                               H  16
      B(5)=1.0                                                          H  17
      RETURN                                                            H  18
      END                                                               H  19

//DATA
```

```
************************
MODULE 2 , FEEDBACK
************************


INPUT DATA
***************

    BETA      LAMBDA      NEUTRON GEN. TIME      INITIAL REACTIVITY
            ( SEC**-1 )       ( SEC )                  ( $ )
  0.006450   0.07695        0.100E-03                  0.30

    INSERTION TIME      CONSTANT PERIOD
        (SEC)               (1/SEC)
         1.000               1.500

    COOLANT COEFF.      CONSTANT B      RESONANCE ESC. PROB.
     (           )      (        )      (                  )
     -0.50000E-04       ********              0.800

    FUEL RADIUS      PITCH      FUEL CP.      FUEL DENSITY      FUEL HEIGHT
      ( FT )         ( FT )  ( BTU/LB-F )   ( LB/FT**3 )        ( FT )
      0.01504        0.04733     0.0590         43.20            12.00

    NUMBER OF ASSEMBLY      NUMBER OF ROD PER ASSEMBLY
    (                 )     (                       )
            145                         208

    COOLANT VELOCITY      INLET COOLANT TEMPT.
       ( FT/SEC )               ( F )
          13.0                  400.0

        OPTION     PLOT          1
                   TYPE INSER.   1

    INITIAL POWER          (MW)  0.1000E 04
    GUESSED FUEL TEMPT.    (F)       500.0
    GUESSED COOLANT TEMPT. (F)       200.0


END OF INPUT DATA
******************



      END TIME     TIME INCREMENT
       8.000          0.0100

     CONVERGENCE FACTOR =    0.01000
```

EQUILIBRIUM STATE INITIALLY
****

POWER	PRECURSOR	FUEL TEMPT.	COOLANT TEMPT.	REACTIVITY
0.1000E 04	0.8382E 06	504.31	412.73	0.000

****

REYNOLDS #	PRANDTL #	COLBURN #	MASS FLOW (LB/HR)	HEAT TRANFER COEFF. (BTU/HR-F-FT**2)
509503.6	1.0	0.042	0.1172E 09	9044.7

NO ( )	TIME (SEC)	REACTIVITY ($)	POWER (MW)	PRECURSOR (MW)
	FUEL TEMP (F)	MOD. TEMP (F)	EXIT TEMP (F)	FUEL COEFF. (1/F)
1	0.000	0.00	0.1000E 04	0.8382E 06
	504.31	412.73	425.47	0.4968E-06
2	0.046	0.30	0.9345E 03	0.8381E 06
	496.42	413.04	426.08	0.5008E-06
3	0.096	0.30	0.1373E 04	0.8379E 06
	497.10	412.92	425.84	0.5004E-06
4	0.146	0.30	0.1420E 04	0.8391E 06
	510.64	413.39	426.79	0.4937E-06
5	0.196	0.29	0.1422E 04	0.8404E 06
	521.65	413.88	427.76	0.4885E-06
6	0.246	0.29	0.1419E 04	0.8418E 06
	529.63	414.32	428.64	0.4848E-06
7	0.296	0.29	0.1416E 04	0.8431E 06
	535.33	414.71	429.42	0.4822E-06
8	0.346	0.28	0.1413E 04	0.8444E 06
	539.41	415.06	430.12	0.4804E-06
9	0.396	0.28	0.1410E 04	0.8457E 06
	542.34	415.37	430.74	0.4791E-06
10	0.446	0.28	0.1408E 04	0.8470E 06
	544.44	415.65	431.30	0.4782E-06
11	0.496	0.28	0.1406E 04	0.8483E 06
	545.97	415.90	431.79	0.4775E-06
12	0.546	0.28	0.1404E 04	0.8496E 06
	547.09	416.12	432.23	0.4770E-06
13	0.596	0.28	0.1403E 04	0.8508E 06
	547.91	416.31	432.62	0.4766E-06
14	0.646	0.27	0.1403E 04	0.8521E 06
	548.53	416.49	432.97	0.4764E-06
15	0.696	0.27	0.1402E 04	0.8533E 06
	549.01	416.64	433.29	0.4762E-06
16	0.746	0.27	0.1402E 04	0.8546E 06

	549.38		416.78	433.57	0.4760E-06	
17		0.796	0.27	0.1402E 04		0.8558E 06
	549.69		416.91	433.82	0.4759E-06	
18		0.846	0.27	0.1402E 04		0.8570E 06
	549.94		417.02	434.05	0.4758E-06	
19		0.896	0.27	0.1402E 04		0.8583E 06
	550.17		417.13	434.26	0.4757E-06	
20		0.946	0.27	0.1403E 04		0.8595E 06
	550.37		417.22	434.44	0.4756E-06	
21		0.996	0.27	0.1403E 04		0.8607E 06
	550.56		417.31	434.61	0.4755E-06	
22		1.046	0.27	0.1404E 04		0.8619E 06
	550.74		417.38	434.76	0.4754E-06	
23		1.096	-0.03	0.1405E 04		0.8631E 06
	550.91		417.45	434.90	0.4753E-06	
24		1.146	-0.03	0.1011E 04		0.8643E 06
	550.95		417.52	435.03	0.4753E-06	
25		1.196	-0.03	0.9980E 03		0.8643E 06
	539.15		417.05	434.10	0.4805E-06	
26		1.246	-0.03	0.1000E 04		0.8642E 06
	530.19		416.62	433.23	0.4845E-06	
27		1.296	-0.03	0.1003E 04		0.8641E 06
	523.75		416.23	432.46	0.4875E-06	
28		1.346	-0.02	0.1005E 04		0.8640E 06
	519.15		415.88	431.77	0.4897E-06	
29		1.396	-0.02	0.1007E 04		0.8639E 06
	515.87		415.58	431.15	0.4912E-06	
30		1.446	-0.02	0.1009E 04		0.8638E 06
	513.54		415.30	430.60	0.4923E-06	
31		1.496	-0.02	0.1011E 04		0.8637E 06
	511.87		415.06	430.11	0.4931E-06	
32		1.546	-0.02	0.1013E 04		0.8637E 06
	510.67		414.84	429.68	0.4937E-06	
33		1.596	-0.01	0.1014E 04		0.8636E 06
	509.82		414.64	429.29	0.4941E-06	
34		1.646	-0.01	0.1016E 04		0.8636E 06
	509.20		414.47	428.95	0.4944E-06	
35		1.696	-0.01	0.1017E 04		0.8635E 06
	508.76		414.32	428.64	0.4946E-06	
36		1.746	-0.01	0.1018E 04		0.8635E 06
	508.43		414.18	428.37	0.4948E-06	
37		1.796	-0.01	0.1019E 04		0.8634E 06
	508.20		414.06	428.12	0.4949E-06	
38		1.846	-0.01	0.1020E 04		0.8634E 06
	508.02		413.95	427.91	0.4950E-06	
39		1.896	-0.01	0.1021E 04		0.8634E 06
	507.89		413.86	427.72	0.4951E-06	
40		1.946	-0.01	0.1021E 04		0.8633E 06
	507.79		413.77	427.55	0.4951E-06	
41		1.996	-0.01	0.1022E 04		0.8633E 06
	507.71		413.70	427.39	0.4952E-06	
42		2.046	-0.01	0.1023E 04		0.8633E 06
	507.66		413.63	427.26	0.4952E-06	
43		2.096	-0.01	0.1023E 04		0.8632E 06
	507.61		413.57	427.14	0.4952E-06	
44		2.146	-0.01	0.1023E 04		0.8632E 06
	507.57		413.52	427.03	0.4952E-06	
45		2.196	-0.01	0.1024E 04		0.8632E 06
	507.54		413.47	426.94	0.4952E-06	
46		2.246	-0.01	0.1024E 04		0.8632E 06

	507.52	413.43	426.85	0.4953E-06		
47	2.296	-0.00		0.1025E 04	0.8632E 06	
	507.50	413.39	426.78	0.4953E-06		
48	2.346	-0.00		0.1025E 04	0.8631E 06	
	507.48	413.35	426.71	0.4953E-06		
49	2.396	-0.00		0.1025E 04	0.8631E 06	
	507.47	413.33	426.65	0.4953E-06		
50	2.446	-0.00		0.1025E 04	0.8631E 06	
	507.46	413.30	426.60	0.4953E-06		
51	2.496	-0.00		0.1025E 04	0.8631E 06	
	507.45	413.28	426.55	0.4953E-06		
52	2.546	-0.00		0.1026E 04	0.8631E 06	
	507.44	413.25	426.51	0.4953E-06		
53	2.596	-0.00		0.1026E 04	0.8631E 06	
	507.43	413.24	426.47	0.4953E-06		
54	2.646	-0.00		0.1026E 04	0.8631E 06	
	507.42	413.22	426.44	0.4953E-06		
55	2.696	-0.00		0.1026E 04	0.8631E 06	
	507.41	413.20	426.41	0.4953E-06		
56	2.746	-0.00		0.1026E 04	0.8630E 06	
	507.41	413.19	426.38	0.4953E-06		
57	2.796	-0.00		0.1026E 04	0.8630E 06	
	507.40	413.18	426.36	0.4953E-06		
58	2.846	-0.00		0.1026E 04	0.8630E 06	
	507.40	413.17	426.34	0.4953E-06		
59	2.896	-0.00		0.1026E 04	0.8630E 06	
	507.39	413.16	426.32	0.4953E-06		
60	2.946	-0.00		0.1026E 04	0.8630E 06	
	507.39	413.15	426.30	0.4953E-06		
61	2.996	-0.00		0.1026E 04	0.8630E 06	
	507.39	413.14	426.29	0.4953E-06		
62	3.046	-0.00		0.1027E 04	0.8630E 06	
	507.38	413.14	426.28	0.4953E-06		
63	3.096	-0.00		0.1027E 04	0.8630E 06	
	507.38	413.13	426.26	0.4953E-06		
64	3.146	-0.00		0.1027E 04	0.8630E 06	
	507.38	413.13	426.25	0.4953E-06		
65	3.196	-0.00		0.1027E 04	0.8629E 06	
	507.38	413.12	426.24	0.4953E-06		
66	3.246	-0.00		0.1027E 04	0.8629E 06	
	507.37	413.12	426.24	0.4953E-06		
67	3.296	-0.00		0.1027E 04	0.8629E 06	
	507.37	413.11	426.23	0.4953E-06		
68	3.346	-0.00		0.1027E 04	0.8629E 06	
	507.37	413.11	426.22	0.4953E-06		
69	3.396	-0.00		0.1027E 04	0.8629E 06	
	507.37	413.11	426.22	0.4953E-06		
70	3.446	-0.00		0.1027E 04	0.8629E 06	
	507.36	413.11	426.21	0.4953E-06		
71	3.496	-0.00		0.1027E 04	0.8629E 06	
	507.36	413.10	426.21	0.4953E-06		
72	3.546	-0.00		0.1027E 04	0.8629E 06	
	507.36	413.10	426.20	0.4953E-06		
73	3.596	-0.00		0.1027E 04	0.8629E 06	
	507.36	413.10	426.20	0.4953E-06		
74	3.646	-0.00		0.1027E 04	0.8629E 06	
	507.36	413.10	426.20	0.4953E-06		
75	3.696	-0.00		0.1027E 04	0.8629E 06	
	507.36	413.10	426.19	0.4953E-06		
76	3.746	-0.00		0.1027E 04	0.8628E 06	

77	507.35	3.796	413.10	-0.00	426.19	0.4953E-06 0.1027E 04 0.8628E 06
78	507.35	3.846	413.09	-0.00	426.19	0.4953E-06 0.1027E 04 0.8628E 06
79	507.35	3.896	413.09	-0.00	426.19	0.4953E-06 0.1027E 04 0.8628E 06
80	507.35	3.946	413.09	-0.00	426.18	0.4953E-06 0.1027E 04 0.8628E 06
81	507.35	3.996	413.09	-0.00	426.18	0.4953E-06 0.1027E 04 0.8628E 06
82	507.35	4.046	413.09	-0.00	426.18	0.4953E-06 0.1027E 04 0.8628E 06
83	507.35	4.096	413.09	-0.00	426.18	0.4953E-06 0.1027E 04 0.8628E 06
84	507.34	4.146	413.09	-0.00	426.18	0.4953E-06 0.1027E 04 0.8628E 06
85	507.34	4.196	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8628E 06
86	507.34	4.246	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
87	507.34	4.296	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
88	507.34	4.346	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
89	507.34	4.396	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
90	507.34	4.446	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
91	507.34	4.496	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
92	507.33	4.546	413.09	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
93	507.33	4.596	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
94	507.33	4.646	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
95	507.33	4.696	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
96	507.33	4.746	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
97	507.33	4.796	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
98	507.33	4.846	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8627E 06
99	507.33	4.896	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
100	507.33	4.946	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
101	507.32	4.996	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
102	507.32	5.046	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
103	507.32	5.096	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
104	507.32	5.146	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
105	507.32	5.196	413.08	-0.00	426.17	0.4953E-06 0.1027E 04 0.8626E 06
106		5.246		-0.00		0.4953E-06 0.1027E 04 0.8626E 06

	507.32	413.08	426.17		0.4954E-06	
107	5.296	-0.00		0.1027E 04		0.8626E 06
	507.32	413.08	426.17		0.4954E-06	
108	5.346	-0.00		0.1027E 04		0.8626E 06
	507.32	413.08	426.17		0.4954E-06	
109	5.396	-0.00		0.1027E 04		0.8626E 06
	507.32	413.08	426.17		0.4954E-06	
110	5.446	-0.00		0.1027E 04		0.8625E 06
	507.32	413.08	426.17		0.4954E-06	
111	5.496	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.17		0.4954E-06	
112	5.546	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.17		0.4954E-06	
113	5.596	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
114	5.646	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
115	5.696	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
116	5.746	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
117	5.796	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
118	5.846	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
119	5.896	-0.00		0.1026E 04		0.8625E 06
	507.31	413.08	426.16		0.4954E-06	
120	5.946	-0.00		0.1026E 04		0.8625E 06
	507.30	413.08	426.16		0.4954E-06	
121	5.996	-0.00		0.1026E 04		0.8625E 06
	507.30	413.08	426.16		0.4954E-06	
122	6.046	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
123	6.096	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
124	6.146	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
125	6.196	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
126	6.246	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
127	6.296	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
128	6.346	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
129	6.396	-0.00		0.1026E 04		0.8624E 06
	507.30	413.08	426.16		0.4954E-06	
130	6.446	-0.00		0.1026E 04		0.8624E 06
	507.29	413.08	426.16		0.4954E-06	
131	6.496	-0.00		0.1026E 04		0.8624E 06
	507.29	413.08	426.16		0.4954E-06	
132	6.546	-0.00		0.1026E 04		0.8624E 06
	507.29	413.08	426.16		0.4954E-06	
133	6.596	-0.00		0.1026E 04		0.8624E 06
	507.29	413.08	426.16		0.4954E-06	
134	6.646	-0.00		0.1026E 04		0.8623E 06
	507.29	413.08	426.16		0.4954E-06	
135	6.696	-0.00		0.1026E 04		0.8623E 06
	507.29	413.08	426.16		0.4954E-06	
136	6.746	-0.00		0.1026E 04		0.8623E 06

	507.29		413.08	426.16		0.4954E-06
137		6.796	-0.00		0.1026E 04	0.8623E 06
	507.29		413.08	426.16		0.4954E-06
138		6.846	-0.00		0.1026E 04	0.8623E 06
	507.29		413.08	426.16		0.4954E-06
139		6.896	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
140		6.946	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
141		6.996	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
142		7.046	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
143		7.096	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
144		7.146	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
145		7.196	-0.00		0.1026E 04	0.8623E 06
	507.28		413.08	426.16		0.4954E-06
146		7.246	-0.00		0.1026E 04	0.8622E 06
	507.28		413.08	426.16		0.4954E-06
147		7.296	-0.00		0.1026E 04	0.8622E 06
	507.28		413.08	426.16		0.4954E-06
148		7.346	-0.00		0.1026E 04	0.8622E 06
	507.28		413.08	426.16		0.4954E-06
149		7.396	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.16		0.4954E-06
150		7.446	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.16		0.4954E-06
151		7.496	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.16		0.4954E-06
152		7.546	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.15		0.4954E-06
153		7.596	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.15		0.4954E-06
154		7.646	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.15		0.4954E-06
155		7.696	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.15		0.4954E-06
156		7.746	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.15		0.4954E-06
157		7.796	-0.00		0.1026E 04	0.8622E 06
	507.27		413.08	426.15		0.4954E-06
158		7.846	-0.00		0.1026E 04	0.8621E 06
	507.27		413.08	426.15		0.4954E-06
159		7.896	-0.00		0.1026E 04	0.8621E 06
	507.26		413.08	426.15		0.4954E-06
160		7.946	-0.00		0.1026E 04	0.8621E 06
	507.26		413.08	426.15		0.4954E-06
161		7.996	-0.00		0.1026E 04	0.8621E 06
	507.26		413.07	426.15		0.4954E-06

RELATIVE DENSITY

```
                0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9
              +-----+-----+-----+-----+-----+-----+-----+-----+-----+
TIME   0.000  |   R                                 P           F  LQM|
TIME   0.046  |                                   P             F L QR|
TIME   0.096  |                                                 F  PQMR
TIME   0.146  |                                                  F  QR|
TIME   0.196  |                                                    F QRL
TIME   0.246  |                                                     FRM|
TIME   0.296  |                                                      RM|
TIME   0.346  |                                                     RFM|
TIME   0.396  |                                                     RQF|
TIME   0.446  |                                                    R QF|
TIME   0.496  |                                                    R QF|
TIME   0.546  |                                                    R QF|
TIME   0.596  |                                                   R    F|
TIME   0.646  |                                                   R    F|
TIME   0.696  |                                                   R    F|
TIME   0.746  |                                                   R    F|
TIME   0.796  |                                                   R    F|
TIME   0.846  |                                                  R     F|
TIME   0.896  |                                                  R     F|
TIME   0.946  |                                                  R     F|
TIME   0.996  |                                                  R     F|
TIME   1.046  |                                                  R     F|
TIME   1.096  |                                                        F|
TIME   1.146  |                                         P            L  F
TIME   1.196  |                                         P            LFM|
TIME   1.246  |                                         P            F M|
TIME   1.296  |R                                        P            F M|
TIME   1.346  |R                                        P           FL M|
TIME   1.396  | R                                       P           FL M|
TIME   1.446  | R                                       P          F L M|
TIME   1.496  | R                                       P          F L M|
TIME   1.546  |  R                                      P          F L M|
TIME   1.596  |  R                                      P          F L M|
TIME   1.646  |  R                                      P          F L M|
TIME   1.696  |  R                                      P          F L M|
TIME   1.746  |   R                                     P          F L M|
TIME   1.796  |   R                                     P          F L M|
TIME   1.846  |   R                                       P        F L M|
TIME   1.896  |   R                                       P        F L M|
TIME   1.946  |   R                                       P        F L M|
TIME   1.996  |   R                                       P        F L M|
TIME   2.046  |   R                                       P        F L M|
TIME   2.096  |   R                                       P        F L M|
TIME   2.146  |    R                                      P        F L M|
TIME   2.196  |    R                                      P        F L M|
TIME   2.246  |    R                                      P        F L M|
TIME   2.296  |    R                                      P        F L M|
TIME   2.346  |    R                                      P        F L M|
TIME   2.396  |    R                                      P        F L M|
TIME   2.446  |    R                                      P        F L M|
TIME   2.496  |    R                                      P        F L M|
TIME   2.546  |    R                                      P        F L M|
TIME   2.596  |    R                                      P        F L M|
TIME   2.646  |    R                                      P        F L M|
TIME   2.696  |    R                                      P        F L M|
TIME   2.746  |    R                                      P        F L M|
```

```
TIME   2.796 |      R
TIME   2.846 |      R
TIME   2.896 |      R
TIME   2.946 |      R
TIME   2.996 |      R
TIME   3.046 |      R
TIME   3.096 |      R
TIME   3.146 |      R
TIME   3.196 |      R
TIME   3.246 |      R
TIME   3.296 |      R
TIME   3.346 |      R
TIME   3.396 |      R
TIME   3.446 |      R
TIME   3.496 |      R
TIME   3.546 |      R
TIME   3.596 |      R
TIME   3.646 |      R
TIME   3.696 |      R
TIME   3.746 |      R
TIME   3.796 |      R
TIME   3.846 |      R
TIME   3.896 |      R
TIME   3.946 |      R
TIME   3.996 |      R
TIME   4.046 |      R
TIME   4.096 |      R
TIME   4.146 |      R
TIME   4.196 |      R
TIME   4.246 |      R
TIME   4.296 |      R
TIME   4.346 |      R
TIME   4.396 |      R
TIME   4.446 |      R
TIME   4.496 |      R
TIME   4.546 |      R
TIME   4.596 |      R
TIME   4.646 |      R
TIME   4.696 |      R
TIME   4.746 |      R
TIME   4.796 |      R
TIME   4.846 |      R
TIME   4.896 |      R
TIME   4.946 |      R
TIME   4.996 |      R
TIME   5.046 |      R
TIME   5.096 |      R
TIME   5.146 |      R
TIME   5.196 |      R
TIME   5.246 |      R
TIME   5.296 |      R
TIME   5.346 |      R
TIME   5.396 |      R
TIME   5.446 |      R
TIME   5.496 |      R
TIME   5.546 |      R
TIME   5.596 |      R
TIME   5.646 |      R
TIME   5.696 |      R
TIME   5.746 |      R
```

```
                                                                P
                                                                P
                                                                P
```

```
TIME   5.796 |    R                                                  P          F L M|
TIME   5.846 |    R                                                  P          F L M|
TIME   5.896 |    R                                                  P          F L M|
TIME   5.946 |    R                                                  P          F L M|
TIME   5.996 |    R                                                  P          F L M|
TIME   6.046 |    R                                                  P          F L M|
TIME   6.096 |    R                                                  P          F L M|
TIME   6.146 |    R                                                  P          F L M|
TIME   6.196 |    R                                                  P          F L M|
TIME   6.246 |    R                                                  P          F L M|
TIME   6.296 |    R                                                  P          F L M|
TIME   6.346 |    R                                                  P          F L M|
TIME   6.396 |    R                                                  P          F L M|
TIME   6.446 |    R                                                  P          F L M|
TIME   6.496 |    R                                                  P          F L M|
TIME   6.546 |    R                                                  P          F L M|
TIME   6.596 |    R                                                  P          F L M|
TIME   6.646 |    R                                                  P          F L M|
TIME   6.696 |    R                                                  P          F L M|
TIME   6.746 |    R                                                  P          F L M|
TIME   6.796 |    R                                                  P          F L M|
TIME   6.846 |    R                                                  P          F L M|
TIME   6.896 |    R                                                  P          F L M|
TIME   6.946 |    R                                                  P          F L M|
TIME   6.996 |    R                                                  P          F L M|
TIME   7.046 |    R                                                  P          F L M|
TIME   7.096 |    R                                                  P          F L M|
TIME   7.146 |    R                                                  P          F L M|
TIME   7.196 |    R                                                  P          F L M|
TIME   7.246 |    R                                                  P          F L M|
TIME   7.296 |    R                                                  P          F L M|
TIME   7.346 |    R                                                  P          F L M|
TIME   7.396 |    R                                                  P          F L M|
TIME   7.446 |    R                                                  P          F L M|
TIME   7.496 |    R                                                  P          F L M|
TIME   7.546 |    R                                                  P          F L M|
TIME   7.596 |    R                                                  P          F L M|
TIME   7.646 |    R                                                  P          F L M|
TIME   7.696 |    R                                                  P          F L M|
TIME   7.746 |    R                                                  P          F L M|
TIME   7.796 |    R                                                  P          F L M|
TIME   7.846 |    R                                                  P          F L M|
TIME   7.896 |    R                                                  P          F L M|
TIME   7.946 |    R                                                  P          F L M|
TIME   7.996 |    R                                                  P          F L M|
             +----+----+----+----+----+----+----+----+----+----+----+----+----+----+
                  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9
INPUT CHARACTER  I#Q FRP1LM
```

P - POWER
L - LOG. OF POWER
Q - PRECURSOR DENSITY
F - FUEL TEMPERATURE
M - MODERATOR TEMPERATURE
R - REACTIVITY
IF VARIABLE PLOTTED HAS NEGATIVE VALUE THE AXIS FOR THIS VARIABLE IS SHIFTED TO THE CENTER OF Y-AXIS